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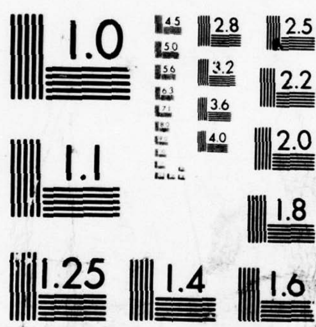

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DESIGN AND ANALYSIS OF SINGLE CELL BALLOONS

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The procedures used in the design and analysis of single cell balloons are reviewed and reformulated to include lobing between load tapes and a realistic model of the load transfer mechanism between load carrying members. Results are presented for a typical "heavy" load balloon which reveal the existence of significant circumferential stresses near the edge of the cap under certain conditions. Results also suggest a more severe state of stress below the design altitude than above.

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"Ah, but a man's reach should exceed his grasp,
Or what's a heaven for?"

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NOMENCLATURE

a - Superpressure at nadir of balloon (feet)

A - Cross-sectional area in horizontal plane $(\text{feet})^2$

b - Specific lift defined as difference in specific weights of atmosphere and inflation gas (pounds/feet^3)

d( ) - Differential quantity as defined

D - Constant used to characterize film behavior as defined by equations [12] and [13] $(\text{Stress})^{-1}$

F - Tensile force carried by an individual load tape (pounds)

K_T - Constant used to characterize tape behavior as defined by equation [14] (pounds)

ℓ - Gore width at any gore position (feet)

N - Number of gores

p - Pressure differential across film (pound/feet^2)

r - Radial location of load tape relative to balloon centerline in a horizontal plane (feet)

R - Radius of curvature of film or tape (feet)

s - Gore position measured from nadir (feet)

T - Total force in the meridional direction divided by 2π (pounds)

V - Volume of the balloon (feet^3)

W - Weight per unit area or length of film or tape respectively $(\text{lbs/ft}^2$ or lbs/ft)

z - Vertical height of an arbitrary point measured from the nadir (feet)

α - Angle in plane containing circumferential radius of curvature defined by equation [28]

β - Angle in plane containing circumferential radius of curvature shown in Figure 3.

γ - Angle defining circumferential radius of curvature shown in Figure 3.

Δ( ) - Symbol denoting a change in quantity

ϵ - Normal strain defined as the change in length per unit length

θ - Angle made by the tangent to the load tape in the meridional direction and the balloon centerline.

σ - Normal stress or stress resultant as defined (pounds/feet^2)

τ - Shear stress or stress resultant as defined (pounds/feet^2)

SUBSCRIPTS

c - Circumferential direction
d - Design Value
f - Film value
i - Arbibrary point in finite difference equation
m - Meridional direction
o - Undeformed value
T - Tape value

I. INTRODUCTION

The design and analysis of free balloons for scientific application has received the attention of many investigators over the years. The pioneering efforts by Upson (1) and the University of Minnesota (2) contributed to the well documented, computerized, shape and stress calculations of Smalley (3,4) in both the fully deployed and partially deployed state. This effort, although completed over a decade ago, continues to be the foundation on which successful balloon designs are generated.

The investigations by manufacturers and research organizations in recent years have been devoted to the application of these procedures to produce various designs, attempts to characterize the film material, or detailed analysis of the state of stress and deviations from the design shape due to material deformation. The most sophisticated of the analysis techniques utilizes finite elements and the minimization of potential energy to obtain deviations from the design shape. Due to the nonlinearities associated with large displacements, the adaptation of such a program (5) to include the loads associated with inflatables is a significant achievement. The effects of lobing have been analyzed by Alexander (6) in a manner which incorporates the characterization of polyethelene films with the design shape. In addition, Rand (7) has utilized finite difference techniques to determine the effects of load introduction into pressurized films.

Many attempts have been made to characterize candidate films and develop the necessary testing apparatus to evaluate their material properties. Considerable effort has been expended in an attempt to evaluate the cold brittleness of a film as a measure of the quality of the material. Both uniaxial and biaxial results have been reported by Weissmann (8), Alexander (9), and Kawada (10), et al. In

addition, the characterization of reinforced films has been studied by Alley (11) and Munson (12). Their results indicate that in the future, composite films may be manufactured with overall properties which are optimum for a particular design.

Each of the studies previously mentioned as well as manufacturing improvement in film and seal quality have been sound engineering programs. As a result many assumptions have been made regarding the shape, stress and failure mechanisms operative in balloon systems. These assumptions have been necessary to obtain operational systems when they were needed. However, Dwyer (13) has attempted to quantify the capability for growth of free balloons in a manner which suggests that the state-of-the-art may have matured to the point of limiting growth. This indicates that there is a need to reassess those assumptions that were made to achieve the current state-of-the-art in the hope of developing an innovative design concept.

At the present time the design process and stress analysis are carried out independently. This is necessitated by the fact that the shape must be known apriori in order to perform any of the analyses previously mentioned. The finite element analysis technique requires the coordinates of each node of every element prior to initiation of the solution scheme. Although very detailed solutions can be obtained by this technique, in order to interface such a technique with an existing design program would require an iterative technique to be developed and excessive amounts of computer time. The analysis technique of Alexander (6) assumes that the shape predicted by the design program is in fact the shape of the gore seams. This is somewhat presumptuous since any limitation of the design shape is automatically included in the stress analysis.

The analysis of Rand (7) in determining the stresses associated with load introduction into a pressurized film involves the simultaneous solution of two partial differential equations in terms of two unknown displacements at each of

200 points. The technique suffers from the same limitations as the finite element routine in that it would require an initial shape, an iterative technique and excessive computer time.

The initial shape calculations on which all analyses are based are not without certain simplifying assumptions which make the problem tractable. By assuming that the meridional stress is uniformly distributed around a symmetrically deployed membrane, the two equations of equilibrium become sufficient for a unique solution. Unfortunately, the stresses are not uniformly distributed throughout the membrane but instead are transferred from the load tapes to the gore panels through a variety of mechanisms. It is presumed that near the apex and nadir of a free balloon in the fully deployed configuration, the meridional loads are carred almost entirely by the load tapes while near the maximum radius of the balloon this load is carried almost entirely by the film. A similar situation will exist during the ascent phase of the flight except that excess material must be carried as payload and the mechanism of load transfer may be altered.

It has been noted by Smalley (14) and others that the capability of placing a specified payload at a specified altitude is limited only by the strength to weight ratio of available films and the ability to seal and handle the film. It should be noted that as efforts continue by the manufacturers to improve the material and handling properties of available films and candidate films, the development of a higher order design procedure may lead to lighter vehicles with the same capabilities as the present generation of vehicles.

II. DESIGN FORMULATION

The normally accepted equations and assumptions for the shape and stresses in a balloon have been thoroughly documented by Smalley (3). The assumptions are as follows:

1. The balloon is rotationally symmetric about a vertical axis.
2. Meridional and circumferential stresses are constant at all points on the circle formed by an intersecting plane normal to the axis of symmetry. This precludes the possibility of shear in the balloon.
3. The densities of the inflation gas and surrounding air are constant.
4. The balloon material is inextensible, thin, and incapable of supporting any bending or compressive loads.

The resulting set of differential equations include two equilibrium equations and four geometric relationships which must be solved simultaneously for the shape and stress variables at each point along the gore. This approach is known in mechanics as the "stress formulation" which is the preferable method when all of the boundary conditions may be expressed in terms of forces rather than displacements. The resulting shape from this formulation is the deformed configuration but, since the film is assumed to be inextensible then the undeformed shape and the manufactured shape will be identical.

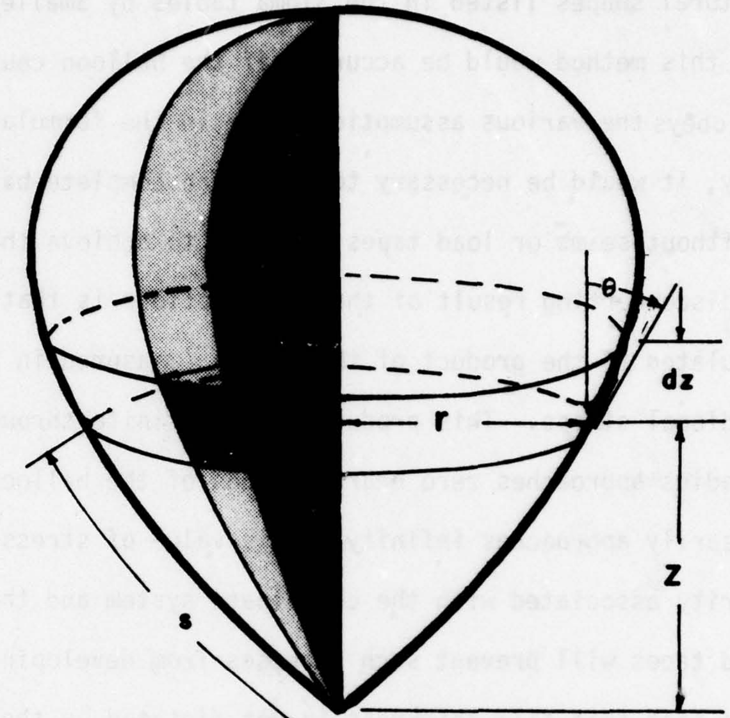
The meridional stress is computed as one of the variables while the circumferential stress must be known in at least some functional form prior to attempting a solution. The most common shape is know as the "natural shape" which is the configuration that results when the circumferential stress is zero. Other designs may be obtained by allowing the circumferential stress to be proportional to the meridional stress, or the change in meridional stress or any other functional relationship. This is an acceptable technique since different shapes will result from

the different assumed stress relationships. The most well know configurations are those natural shapes listed in the Sigma tables by Smalley (3). The stresses predicted by this method would be accurate if the balloon could be built in a manner which obeys the various assumptions made in the formulation of the equations. Unfortunately, it would be necessary to build the complete balloon as a shell of revolution without seams or load tapes in order to achieve this shape and stress.

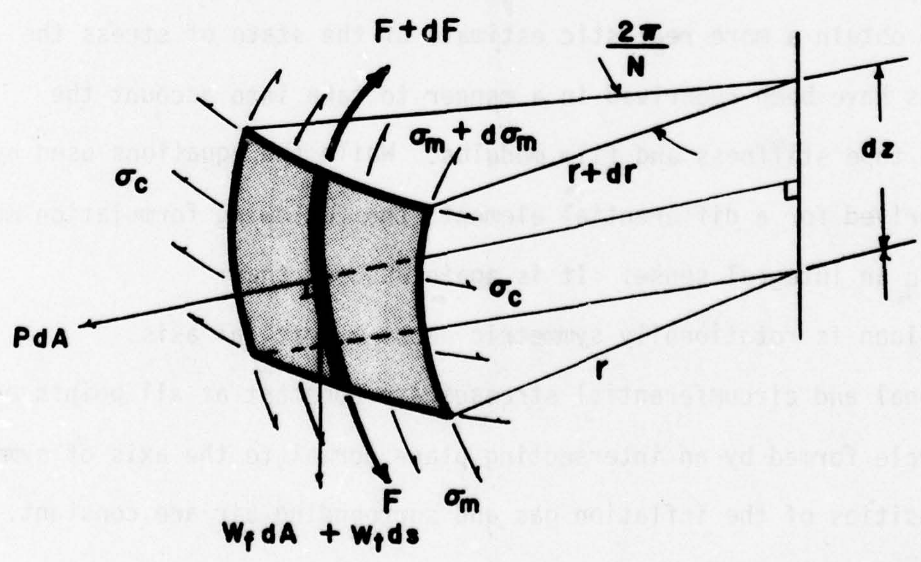
Another disconcerting result of these assumptions is that the meridional film load is formulated as the product of the radius (measured in the horizontal plane) and the meridional stress. This product remains finite throughout the balloon but as the radius approaches zero near the ends of the balloon, the meridional stress necessarily approaches infinity. This value of stress is usually ignored as a singularity associated with the coordinate system and the practical realization that the load tapes will prevent such stresses from developing. Another mitigating factor is the fact that film thickness is not dictated by the stress predicted by this procedure but by practical manufacturing limitations. However, as balloons become larger and payloads become heavier, the normally safe designs may become marginal.

In order to obtain a more realistic estimate of the state of stress the design equations have been rederived in a manner to take into account the effects of load tape stiffness and film modulus. While the equations used by Smalley were derived for a differential element, the following formulation must be considered in an integral sense. It is again assumed that:

1. The balloon is rotationally symmetric about a vertical axis.
2. Meridional and circumferential stresses are constant at all points on the circle formed by an intersecting plane normal to the axis of symmetry.
3. The densities of the inflation gas and surrounding air are constant.
4. Both film and tape material are linearly elastic and orthotropic.



(a) - Balloon Coordinates



(b) Forces Acting on Dicrete Element

Figure 1 - Discrete Balloon Design Element

5. The meridional strain in the film is equal to the meridional strain in the tape.

The discrete element is shown in Figure 1 and represents a portion of the surface bounded by two horizontal planes separated by a distance dz and two vertical planes containing the axis of symmetry and the centerlines of two adjacent gores. The angle separating these two planes is then $2\pi/N$ where N is the number of gores. By summing forces in the meridional direction it can be shown that:

$$\frac{2\pi}{N}d(r\sigma_m) + dF = \frac{2\pi}{N}\sigma_c \sin\theta ds + (\frac{2\pi}{N} rW_f + W_T)\cos\theta ds$$

Therefore, $$\frac{d(r\sigma_m + \frac{NF}{2\pi})}{ds} = \sigma_c \sin\theta + (rW_f + \frac{NW_T}{2\pi})\cos\theta \qquad [1]$$

After summing forces perpendicular to the element the following equation results:

$$(r\sigma_m + \frac{NF}{2\pi})\frac{d\theta}{ds} = \sigma_c \cos\theta - (rW_f + \frac{NW_T}{2\pi})\sin\theta - pr \qquad [2]$$

If the meridional load parameter is defined as:

$$T \equiv r\sigma_m + \frac{NF}{2\pi} \qquad [3]$$

And if the distributed weight parameter is defined as:

$$rW \equiv rW_f + \frac{NW_T}{2\pi} \qquad [4]$$

Then equations [1] and [2] may be rewritten in the following form:

$$\frac{dT}{ds} = \sigma_c \sin\theta + rW\cos\theta \qquad [5]$$

and $$T\frac{d\theta}{ds} = \sigma_c \cos\theta - rW\sin\theta - br(z + a) \qquad [6]$$

Where the pressure differential is given as usual by:

$$p \equiv b(z + a) \qquad [7]$$

The usual geometric relations are:

$$\frac{dr}{ds} = \sin\theta \qquad [8]$$

$$\frac{dz}{ds} = \cos\theta \qquad [9]$$

$$\frac{dA}{ds} = 2\pi r \qquad [10]$$

$$\frac{dV}{ds} = \pi r^2 \cos\theta \qquad [11]$$

In this form the set of equations is identical to those used by Smalley; therefore, any existing design may be used to obtain the variables in question, i.e., T, θ, r, z, A, and V. However, the difference between this procedure and previous approaches to the problem is in the evaluation of the state of stress and the manufactured geometry of the balloon.

III. DESIGN PROGRAM DEVELOPMENT

The essential difference between the usual design procedure and the procedure described here lies in the interpretation of the final results. The total meridional load, equation [3], must be partitioned between the film and tapes according to their respective material properties. When this is done, the resulting film stresses may be used in conjunction with the appropriate constitutive equation for the film to determine the undeformed or stress free shape of the balloon. This is the shape that should be used for manufacturing purposes instead of the deformed shape as is presently used.

If it is assumed that the film is linearly elastic and orthotropic, then the governing constitutive equations may be written as:

$$\varepsilon_m = D_m \sigma_m + D_{mc} \sigma_c \qquad [12]$$

$$\varepsilon_c = D_{mc} \sigma_m + D_c \sigma_c \qquad [13]$$

It should be recalled that the circumferential stress is known as an input to the design process. As a common example, the fully tailored natural shape design assumes σ_c to be zero. Equation [3] may now be rewritten assuming that the meridional strain in the film and tape are equal.

$$\sigma_m = \frac{T}{\left(r + \frac{N K_T D_m}{2\pi}\right)} \qquad [14]$$

Here K_T is the tape modulus and relates the tape force to the tape strain in a linear fashion.

The undeformed shape of the balloon may now be found from the definition of strain. In particular, the original radius of the balloon at any gore position is found from the circumferential strain:

$$\varepsilon_c \equiv \frac{\Delta r}{r_o} = \frac{r}{r_o} - 1 \qquad [15]$$

as $r_o = \frac{r}{(1 + \varepsilon_c)}$ [16]

where ε_c is found from equations [13] and [14]. The undeformed gore position is determined from the definition of meridional strain. In particular:

$$\varepsilon_m \equiv \frac{\Delta(ds)}{ds_o} = \frac{ds}{ds_o} - 1 \qquad [17]$$

Therefore

$$ds_o = \frac{ds}{(1 + \varepsilon_m)} \qquad [18]$$

The original gore position is then found by itegration of equation [18] to any deformed position, s , along the gore.

In order to perform these computations it is necessary to have an efficient routine design program that will accomodate a variety of films, tapes and flight conditions. Therefore, a balloon design program was written on the basis of the "BALLOON" and "FLATBALL" programs reported by Smalley (3). These programs were modified somewhat to include an automatic computation of the starting angle which significantly speeds convergence. The program consists of a variety of subroutines which eliminates the need to manually compute a variety of parameters needed for the design process. A listing of this program is contained in Appendix A with sufficient documentation to permit use of the program.

The various subroutines perform the following functions:

A. BALDE - This is an executive routine which reads the required input information on the type of balloon to be designed. After the design has been established it computes the load-altitude curve of the balloon.

B. SUBROUTINE DESIGN - This routine is based on the computational scheme reported by Smalley (3). It is modified to compute the various stresses in accordance with equation [14] and computes the balloon table layout on the basis of equations [16] and [18].

C. SUBROUTINE MATL - This routine computes the various material properties needed by the design process such as film and tape stiffness (modulus) and weight. The computations are based on data from a variety of sources and are empirically fitted functions of temperature.

D. SUBROUTINE ATMOS - This is a program for computing the various standard properties and specific lift when the design altitude is given in meters.

E. SUBROUTINE ATMOS.2 - This program performs the same function as SUBROUTINE ATMOS except the design altitude is given in millibars of pressure.

F. SUBROUTINE BOYNCY - This routine uses the standard atmosphere to compute the pressure altitude when the specific bouyancy is known. It is used in the computation of the load-altitude curve.

G. SUBROUTINE GLNGTH - This is an empirical fit of the unique gore length-gross load curve reported by Smalley (15).

IV. DESIGN RESULTS

In order to demonstrate the impact of this procedure a number of studies have been performed which demonstrate the influence of various parameters such as film thickness and number of load tapes on the film stress. One such study has been presented by Keese (16) where he concluded that:

A. The total surface area of a balloon and its manufactured cost may be significantly reduced if the design were based on a film stress including load tape effects.

B. The total surface area of a balloon and its manufactured cost may be modestly reduced by simultaneously increasing the number of load tapes and reducing the film thickness.

Although these conclusions may have been premature, the existence of a tolerable state of stress is predicted under design conditions.

As a second example of this program, a balloon was designed which has a very similar set of characteristics as a 20.8 MCF balloon used by Alexander (6) to study the effects of lobing. The balloon was designed as a fully tailored natural shape with the following design parameters:

Design Altitude:	126,000 ft.
Design Payload:	2,700 lbs.
Maximum Payload:	5,250 lbs.
Film Material:	Polyethelene
Shell Thickness:	.0008 in.
Cap Thickness:	.0017 in.
Load Tape Material:	Polyester
Load Tape Rating:	400 lbs.
Inflation Gas:	Helium

Sample input and output for this particular balloon are given in Appendix B. The integration step DSO was adjusted until the final increment of integration at the top of the balloon was approximately equal to all other steps. The resulting design contained 202 points (201 increments) and has the following characteristics:

Balloon Volume: 20.9 MCF

Surface Area: 375,000 $ft.^2$

Film Weight: 2214 lbs.

Tape Weight: 561 lbs.

Deformed Gore Length: 530.2 ft.

Undeformed Gore Length: 527.4 ft.

Deformed Half Gore Width: 50.60 in (maximum)

Undeformed Half Gore Width: 50.72 in (maximum)

It may be noted that the maximum difference between the deformed and undeformed half gore width is .12 inch which is probably not within manufacturing tolerances. However, this small difference may cause a strain of 0.2% in the circumferential direction. It may also be noted that this balloon will elongate 2.8 feet from its manufactured length when loaded under design conditions. However, by building the balloon according to its deformed length, excess material is included in the meridional direction. This feature may serve to reduce the strain in that direction.

The meridional stress predicted by the usual assumption of a tapeless construction is shown for comparison purposes with that stress predicted by equation [14] in Figure 2 in non-dimensional form. It may be noted that the film stress is significantly less than the stress normally assumed and always remains finite. The balance of the forces in the meridional direction is carried by the load tapes under these conditions. However, in both cases the circumferential stresses are zero which is an inherent assumption in the natural shape design.

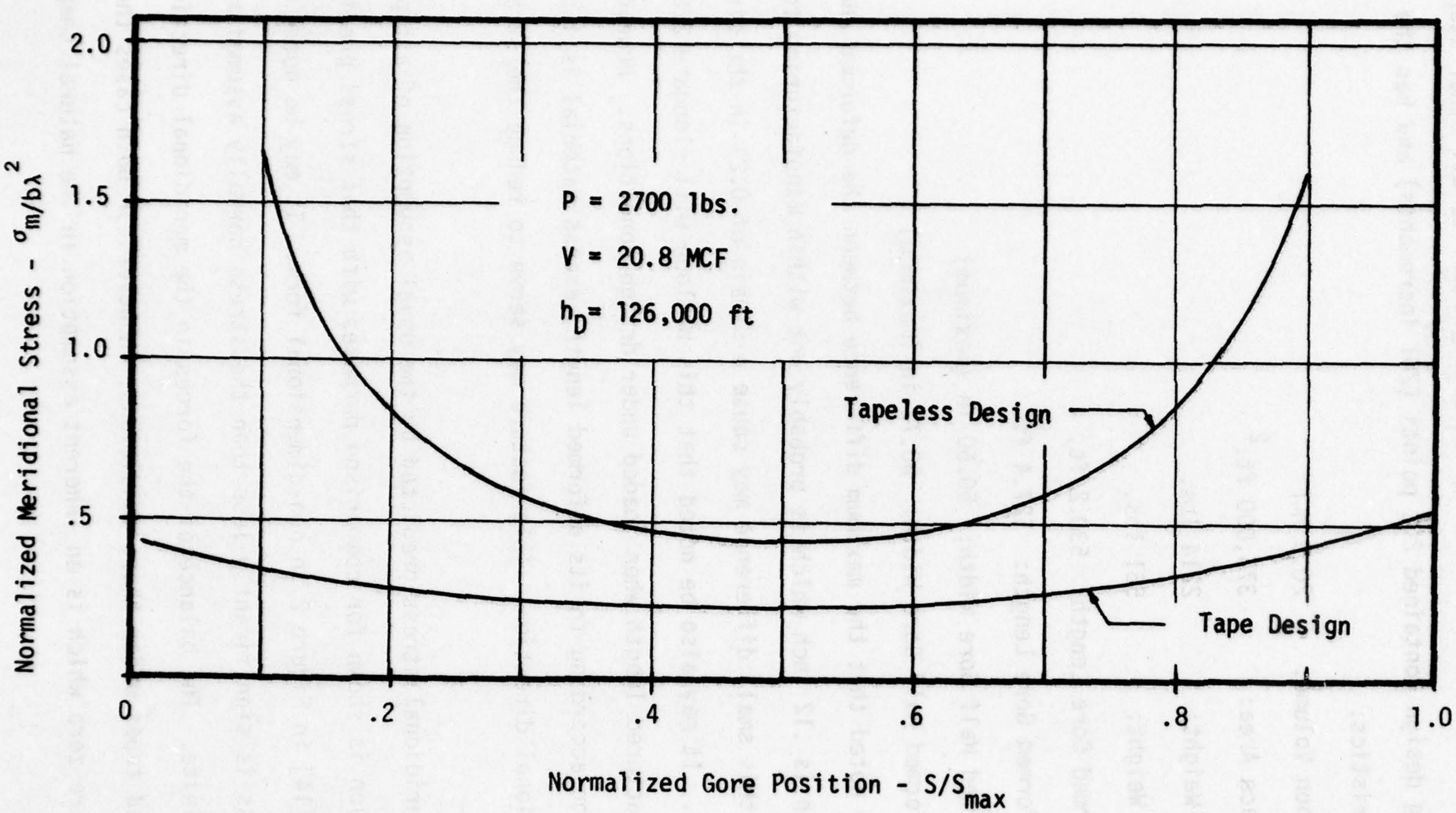


Figure 2 - Meridional Stress Distribution

V. ANALYSIS FORMULATION

It has been shown that the inclusion of material properties is relatively straight forward provided the design shape is assumed to be correct. However, the stress analysis of a flexible structure is significantly different from the design problem. Once the balloon has been manufactured, the available material at each gore position is fixed and the solution should be obtained as a boundary value problem rather than as an initial value problem. In addition, the simplifying assumptions of zero circumferential stress and a surface of revolution should not be made since it is suspected that this will significantly alter the state of stress. This problem has been recognized for many years and various researchers have obtained solutions with varying degrees of success.

Gilbert (17) was able to successfully model a parachute structure utilizing a global transformation matrix to relate the local coordinates to generalized coordinates. However, in order to optimize the aerodynamic drag characteristics of the system, he simplified his model to the case of zero circumferential stress which would correspond to the design shape of a balloon system. Although the formulation of the differential equations is similar to the approach to be presented here, the simplifying assumptions and radically different boundary conditions prevent the use of this program. Alexander solved this problem for several balloon systems by assuming that the load tapes assumed the design shape while the film was permitted to bow out between the tapes. He permitted the meridional radius of curvature to change across the gore but this in turn prevented the equilibrium equation in the meridional direction from being satisfied.

The formulation to be presented here will postulate a mechanism by which loads are transfered between the load tapes and the film. The combination of shear stress and lobing between the load tapes will permit the coupling of the tape shape to the film shape. In the process of developing this formulation a number of features will be introduced for the first time. In order to eliminate as much confusion as possible, Lagrangian coordinates will be used whenever possible.

Lagrangian coordinates are routinely used in time dependent problems in the area of fluid and solid mechanics to denote the position of the particles initially. However, this system of coordinates is ideally suited to problems in elasticity where material properties have been defined with respect to the undeformed dimensions. All balloon material properties now being developed at Texas A&M University utilize "engineering" stress and strain rather than "true" stress and strain. Therefore, in this system of coordinates, the stress is defined as the load per unit initial (as manufactured) area rather than the actual area. The Lagrangian gore length remains the manufactured length, regardless of the actual deformed length of the gore. Another important quantity which is simplified by this technique is the increment of balloon film or tape weight between any two gore positions. This increment is known from the manufactured shape and remains fixed regardless of deformation.

In general, it will be assumed that each balloon gore deploys in an identical manner about the balloon centerline. All deformations will be assumed to be linearly elastic and orthotropic although numerical results have been obtained only for the isotropic case. The film will be permitted to lobe between the load tapes resulting in a surface which is not a surface of revolution. Due to the assumed symmetry of deployment, equilibrium equations will be written for a single differential element of tape and an adjacent element of film at an arbitrary gore position.

The usual deformed balloon coordinate description as shown in Figure 3 will be used to demonstrate the compatibility of this formulation with the familiar design equations. The film is assumed to lobe in a plane containing the meridional radius of curvature to two adjacent load tapes. The deployed distance of the load tape from the centerline in the horizontal plane is designated r.

Consider a differential length of tape at some arbitrary point on the balloon as shown in Figure 4. The forces acting on this element include a changing tape force, F, as well as forces due to the film stresses, τ and σ_c, and the tape weight. It may be noted at this point that the actual length of the element is ds whereas the original length was ds_o. Since "engineering" stress will be used consistently, the original film area will be $t_o ds_o$ where t_o is the undeformed film thickness. Therefore, the stress will always be multiplied by t_o so that the units of stress may simply be considered to be load per unit original length.

Summing forces in the vertical direction it can be shown that:

$$(F+dF)\cos(\theta+d\theta) - F\cos\theta - W_T ds_o - 2\tau\cos\theta\, ds_o - 2\sigma_c \sin\beta \sin\theta\, ds_o = 0$$

The circumferential stress term arises from the fact that β is defined in a plane containing the meridional radius of curvature and not the horizontal plane. This equation may be rewritten by taking the limit as ds_o and $d\theta$ approach zero as:

$$\frac{d(F\cos\theta)}{ds_o} = W_T + 2\tau\cos\theta + 2\sigma_c \sin\beta \sin\theta \qquad [19]$$

A second differential equation may be obtained by summing forces in the horizontal plane. The resulting differential equation after taking the limit as above becomes:

$$\frac{d(F\sin\theta)}{ds_o} = 2\tau\sin\theta - 2\sigma_c \sin\beta \cos\theta \qquad [20]$$

Equations [19] and [20] express the equilibrium of forces acting at any point along the load tape.

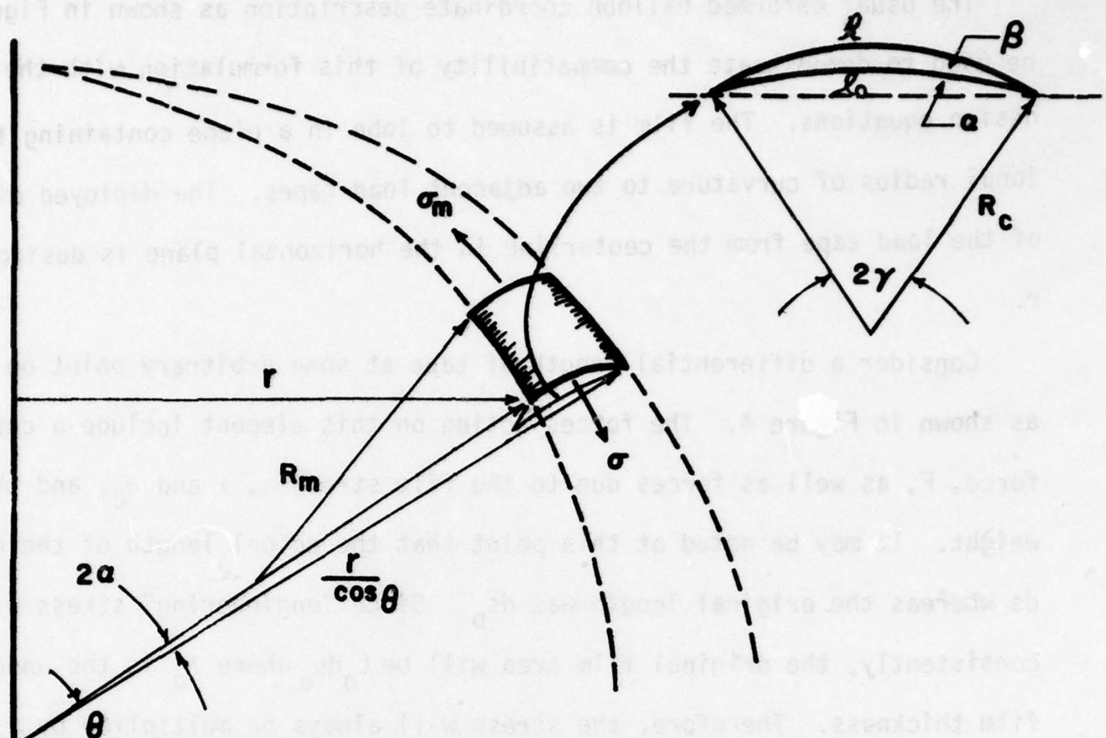


Figure 3 - Discrete Balloon Analysis Element

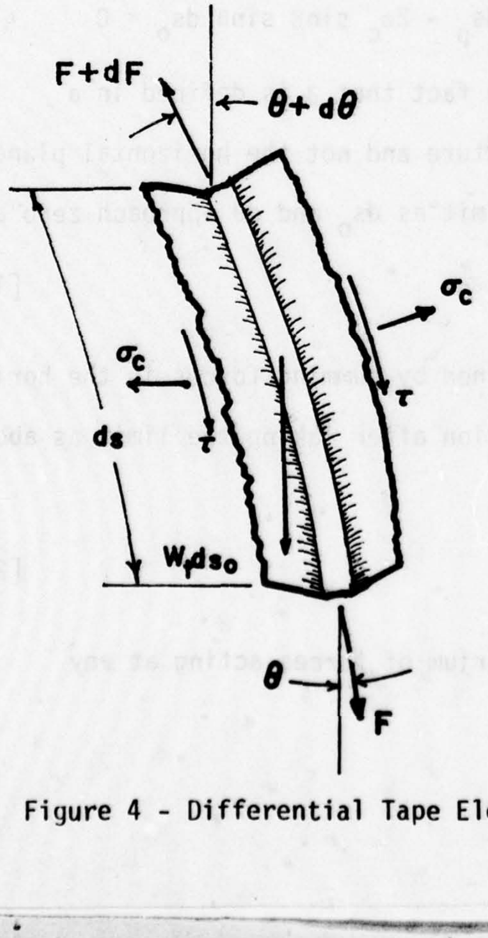


Figure 4 - Differential Tape Element

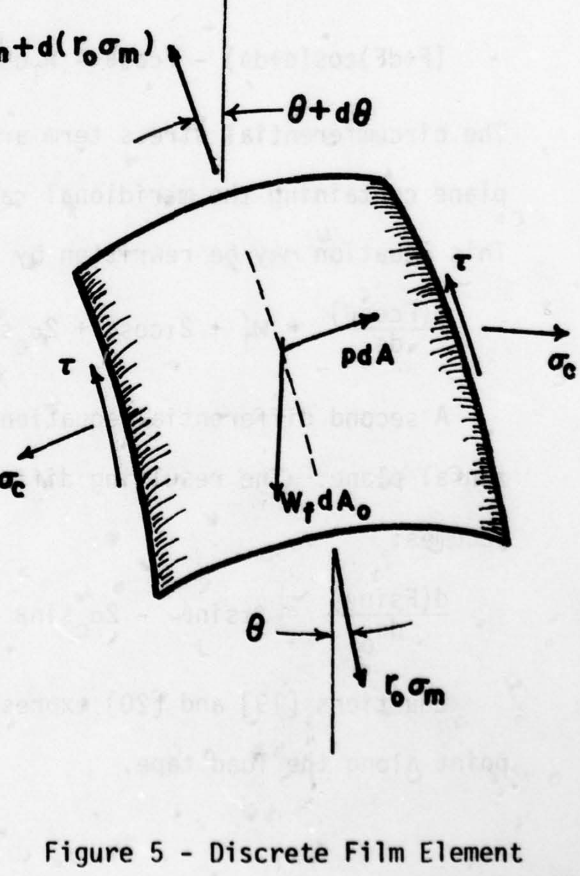


Figure 5 - Discrete Film Element

Consider now a differential length of film that runs between two adjacent load tapes at the same gore position as shown in Figure 5. Here it is assumed that a constant meridional stress is uniformly distributed across the gore width which is $2\pi r_o/N$ where N is the number of gores and r_o is the manufactured radius of the balloon. The meridional stress is permitted to change in the meridional direction as shown. In addition there will be forces due to the shear stress, τ, the circumferential stress, σ_c, the film weight which is expressed in Lagrangian coordinates and the force due to pressure. This last force is obtained by using the actual projected area which may then be expressed in the original coordinates by observing the definition of meridional strain, i.e.,

$$\frac{ds}{ds_o} = 1+ \varepsilon_m \qquad [17]$$

Summing forces in the vertical direction it can be shown that:

$$\frac{2\pi}{N}(r_o\sigma_m+d(r_o\sigma_m))\cos(\theta+d\theta) - \frac{2\pi}{N}r_o\sigma_m\cos\theta - \frac{2\pi}{N}r_oW_f ds_o + 2\tau\cos\theta\, ds_o$$

$$-2pr\sin(\tfrac{\pi}{N})\sin\theta\, ds + 2\sigma_c\sin\beta\,\sin\theta\, ds_o = 0$$

Utilizing equation [17] and taking the limit as ds_o and $d\theta$ approach zero, this equation becomes:

$$\frac{d(r_o\sigma_m\cos\theta)}{ds_o} = r_oW_f + pr(1+\varepsilon_m)\frac{N}{\pi}\sin(\tfrac{\pi}{N})\sin\theta - \frac{N\tau}{\pi}\cos\theta - \frac{N}{\pi}\sigma_c\sin\beta\,\sin\theta \qquad [21]$$

A second differential equation for the film may be obtained by summing forces in the horizontal plane and taking the limit as before. The resulting equation is:

$$\frac{d(r_o\sigma_m\sin\theta)}{ds_o} = -pr(1+\varepsilon_m)\frac{N}{\pi}\sin(\tfrac{\pi}{N})\cos\theta - \frac{N\tau}{\pi}\sin\theta + \frac{N}{\pi}\sigma_c(\sin(\tfrac{\pi}{N})\cos\beta + \cos\tfrac{\pi}{N}\sin\beta\cos\theta) \qquad [22]$$

In working with these equilibrium equations it is a simple matter to rotate the system in such a way that the equilibrium equations in the meridional and normal directions are obtained. When this is accomplished the equations become:

Meridional Tape Equilibrium

$$\frac{dF}{ds_o} = 2\tau + W_T\cos\theta \quad [23]$$

Normal Tape Equilibrium

$$F\frac{d\theta}{ds_o} = -(W_T\sin\theta + 2\sigma_c\sin\beta) \quad [24]$$

Meridional Film Equilibrium

$$\frac{d(r_o\sigma_m)}{ds_o} = r_oW_f\cos\theta - \frac{N}{\pi}\tau + \frac{N}{\pi}\sigma_c\ \sin\theta\ [\sin\frac{\pi}{N}\ \cos\beta + \cos\theta\ \sin\beta\ (\cos\frac{\pi}{N} - 1)] \quad [25]$$

Normal Film Equilibrium

$$r_o\sigma_m(\frac{d\theta}{ds_o}) = -r_oW_f\sin\theta - pr(1+\epsilon_m)\frac{N}{\pi}\ \sin\frac{\pi}{N} + \frac{N}{\pi}\sigma_c[\sin\frac{\pi}{N}\ \cos\beta\ \cos\theta$$

$$+ \sin\beta(\cos\frac{\pi}{N}\cos^2\theta + \sin^2\theta)] \quad [26]$$

It may be noted that the governing equations in the normal direction, equations [24] and [26], are not influenced by the shear stress. However, at this point no distinction has been made between the angle θ when referred to the tape or the film. A discussion of this critical feature will be deferred to the next section of this report.

Several additional equations are required to form a complete set. If the circumferential strain is defined as the change in gore width per unit original gore width, it may be expressed as:

$$1+\epsilon_c = \frac{r\ \sin\pi/N}{r_o\sin(\alpha+\beta)}\ \frac{(\alpha+\beta)}{\pi/N} \quad [27]$$

This equation is obtained by considering Figure 3. The deformed gore width as shown here is:

$$\ell = 2R_c\gamma$$

where R_c is the circumferential radius of curvature and $\gamma = \alpha+\beta$. The undeformed gore width is obtained from the manufacturedshape, i.e.,

$$\ell_o = \frac{2\pi r_o}{N}$$

where r_o is the design value of balloon radius at any point.

The angle α is obtained by noting that the distance between load tapes in the horizontal plane is identical to the distance between load tapes in a plane containing the circumferential radius of curvature, R_c. Then:

$$\frac{r}{\cos\theta}\sin\alpha = r\sin\frac{\pi}{N}$$

$$\text{or } \alpha = \sin^{-1}\left[\sin\frac{\pi}{N}\cos\theta\right] \qquad [28]$$

By the same logic:

$$R_c\sin\gamma = r\sin\frac{\pi}{N} \qquad [29]$$

Equation [27] may then be obtained from the definition of circumferential strain as:

$$\varepsilon_c \equiv \frac{\ell-\ell_o}{\ell_o}$$

The material is assumed to be linearly elastic and orthotropic so that Equations [12] and [13] are still applicable. Finally, the geometric relationships are employed such that:

$$\frac{dr}{ds_o} = (1+\varepsilon_m)\sin\theta \qquad [30]$$

and

$$\frac{dz}{ds_o} = (1+\varepsilon_m)\cos\theta \qquad [31]$$

It is now a matter of arranging these equations into a suitable form which will be tractable for solution.

VI. ANALYSIS PROGRAM DEVELOPMENT

In order to solve the set of equations just developed it is necessary to make some additional assumptions regarding the stress field and the tape angle. Since the tape force, F, appears in both tape equilibrium equations it is necessary to employ some sort of relationship between this force and the film forces. Therefore, it is assumed that the tape force is proportional to the meridional strain in the film. Therefore,

$$F = K_T\varepsilon_m = K_T D_m \sigma_m + K_T D_{mc} \sigma_c \quad [32]$$

Although this assumption is reasonable at the interface between the tape and film, the assumption of a uniformly distributed stress across the gore will result in a uniform strain which will dictate the tape force. The alternative to this approximation is to allow the meridional strain to vary across the gore. This would result in a set of partial differential equations which would increase the computational time considerably. Another approach would be to assume a strain distribution across the gore and use the integral of the resulting stress distribution. An approach similar to this was used by Alexander (6).

The equations of equilibrium may be arranged in a more familiar form if the total meridional load is defined as the sum of the tape forces and the film forces. Specifically,

$$T \equiv r_o \sigma_m + \frac{NF}{2\pi} \quad [33]$$

The two meridional equilibrium equations [23] and [25] may be added together to finally yield,

$$\frac{dT}{ds_o} = \left(\frac{NW_T}{2\pi} + r_o W_f\right)\cos\theta + \frac{N}{\pi}\sigma_c \sin\theta\left[\sin\left(\frac{\pi}{N}\right)\cos\beta + \sin\beta\cos\theta\left(\cos\frac{\pi}{N} - 1\right)\right] \quad [34]$$

It may be noted that the shear stress which appeared in both of the contributing equations is self equilibrating and does not appear in equation [34]. In addition

if the circumferential stress is assumed to be zero, this equation will reduce to the design equation for a natural shape balloon as reported by Smalley.

In a similar manner the equilibrium equations in the normal direction, equations [24] and [26] may be combined. However, in doing so it is necessary to assume that the tape and film are deployed at the same angle. When added together the following equation is obtained:

$$T\frac{d\theta}{ds_o} = -\left(\frac{NW_T}{2\pi} + r_o W_f\right)\sin\theta - pr(1+\epsilon_m)\frac{N}{\pi}\sin\frac{\pi}{N} + \frac{N}{\pi}\sigma_c\left[\sin\frac{\pi}{N}\cos\beta\cos\theta + \right.$$

$$\left.\sin\beta\left(\cos\frac{\pi}{N}\cos^2\theta - \cos^2\theta\right)\right] \qquad [35]$$

This equation will also reduce to the familiar design equation if there is no circumferential stress. It must be observed that in order to obtain this equation no distinction may be made between the tape and film angles. This approximation is quite reasonable over the majority of the gore length; however, near the ends θ must be thought of as the average of two different angles.

A third useful relation may be obtained if it is assumed that the derivatives in equations [24] and [26] are equal. If the difference between these two derivaties is set equal to zero then the following algebraic equation will result:

$$r_o W_f \sin\theta + pr(1+\epsilon_m)\frac{N}{\pi}\sin\frac{\pi}{N} - \frac{N}{\pi}\sigma_c\left[\sin\frac{\pi}{N}\cos\beta\cos\theta + \sin\beta\left(\cos\frac{\pi}{N}\cos^2\theta + \sin^2\theta\right)\right]$$

$$-\frac{N}{2\pi}\frac{r_o\sigma_m}{(1-r_o\sigma_m)}\left(W_T\sin\theta + 2\sigma_c\sin\beta\right) = 0 \qquad [36]$$

This equation places a very severe limitation on the shape of the deployed balloon since it effectively requires the tape angle to be identical to the film angle. Although this assumption appears to be reasonably valid over most of the gore length, it is obviously in error near the end points. However, since this is an algebraic equation rather than a differential equation, it may be used to determine the lobing angle for any set of shape variables without integration.

This equation has no counterpart in the design procedure since no distinction is made between the tape and film angles.

Since equations [34], [35], and [36] do not involve the shear stress, this variable need not be evaluated to determine the shape. However, once the shape is found equation [23] may be used to evaluate the shear stress and ultimately the principal stresses and tension field patterns.

In order to obtain solutions at a variety of altitudes, it was decided to include a volume calculation. This is necessary to provide an overall boundary condition on the problem. Since the total volume is needed at the bottom of the balloon to establish equilibrium with the payload it was decided to integrate the governing differential equation from top to bottom with the boundary condition of zero volume at the apex of the balloon. All other differential equations will be integrated from bottom to top so that for a positive change in gore position, the change in volume will be negative. The governing differential equation may be simply stated as:

$$\frac{dV}{ds_o} = (1+\varepsilon_m)\frac{dV}{ds} = -(1+\varepsilon_m)A\cos\theta \qquad [37]$$

In this case, A is the area in a horizontal plane which is obtained by multiplying the area enclosed by a single gore by the number of gores. The area in question is composed of two parts; a) the triangle formed by straight lines in the horizontal plane connecting the two adjacent load tapes and the centerline; and b) the projection of the lobe onto the horizontal plane. This second area will be positive or negative depending on whether or not the pressure differential is positive or negative. It may be shown by considering Figure 3 that the area of a single gore in the horizontal plane is given by:

$$A = \frac{r^2}{2}\left\{\sin^2\left(\frac{\pi}{N}\right) \pm \cos\theta\,\frac{\sin^2\left(\frac{\pi}{N}\right)[2(\alpha+\beta)-\sin(2\alpha+2\beta)]}{2\sin^2(\alpha+\beta)}\right\} \qquad [38]$$

The choice of the proper sign to be used in this equation is made by dividing the pressure by the absolute value of the pressure.

Equations [30] through [37] must be solved simultaneously in order to insure a compatible shape. An attempt was made to solve this set of equations utilizing the same modified Runge-Kutta numerical technique which is routinely used for the design of balloon shapes. However, this technique is best suited for initial value problems rather than boundary value problems. This so called "shooting" method is unable to impose a final boundary condition and must rely on the proper adjustment of initial conditions until the desired solution is obtained at the final boundary. In this particular case, the solution of five coupled differential equations is stable for only certain values of circumferential stress. As sufficiently large stresses are developed the integration scheme becomes unstable, diverges and a solution is unobtainable. It is for this reason that limits are placed on designs with circumferential stress. It must be emphasized that this is a difficulty with the numerical technique and not with the formulation of the problem.

In an effort to obtain a stable solution with implicit control of the boundary conditions, the governing differential equations were rewritten in finite difference form to form a set of nonlinear algebraic equations. The balloon gore is divided into a large number of points and the six equations written at each point. This results in a set of equations which can be efficiently handled due to the banded nature of the problem. A Newton-Raphson technique was then attempted but did not yield a convergent solution. It is difficult to ascertain the reason for this lack of convergence but it is suspected that the derivatives of the non-linear function defined by equation [31] near the top of the balloon approach zero which cause the coefficient matrix to become singular.

A direct iteration technique was employed to eliminate the instability associated with the inversion of a singular matrix. In this technique an

equation is written for each variable in terms of known quantities. For example, equation [30] may be used to evaluate the radius at point i, r_i, for the next iteration step according to the equation:

$$r_i = r_{i+1} - \Delta s_o (1+\varepsilon_m)_i \sin\theta_i$$

Similar equations are written for each variable at every point. The right hand side of each equation is evaluated from current values and the left hand side is used in the next iteration step on the right hand side until the changes are suitably small. As in the case of the Newton-Raphson technique, the direct iteration technique was found to be divergent when applied directly. However, by using an under relaxation technique where only a portion of the changes called for in each variable were used, the solution became stable and convergent. When the design shape is used for the initial shape estimate, convergence is obtained in 13 iterations. An intermediate step in the iteration process is the computation of meridional and circumferential film stresses at each point along the gore. This is accomplished by solving equations [32] and [33] simultaneously with the membrane equation applied at the center of each gore, i.e.,

$$\frac{\sigma_m}{R_m} + \frac{\sigma_c}{R_c} = p + W_f \sin\theta \qquad [39]$$

The angle θ is positive when measured clockwise from the vertical parallel to the balloon centerline. As a result, since s is measured positive from the nadir, the meridional radius of curvature will be positive if the change is angle is negative. Therefore:

$$\frac{1}{R_m} = -\frac{d\theta}{ds_o}\frac{1}{(1+\varepsilon_m)}$$

In equation [39] the weight per unit original area is assumed to be negligibly different from the weight per unit actual area.

VII. ANALYSIS PROGRAM RESULTS

The analysis program was applied to the balloon described in Section IV of this report. Appendix B contains a detailed description of the initial shape and weight distributions as well as the deformed parameters given by the usual design process. This particular balloon was selected for analysis because it is very similar to the balloon analyzed by Alexander (6) in attempting to determine the circumferential stresses due to lobing. In addition, this balloon is typical of the "heavy" load balloons which have experienced an inordinate number of failures in recent months. The problem was first solved for the design altitude so that the results could be compared to those of Alexander. It was then assumed that the balloon was in equilibrium at a variety of altitudes both above and below the design altitude. The altitude regime below the design altitude was selected to correspond to those altitudes that would have been recommended by the manufacturer on the basis of cap thickness and load tape strength. A convergent solution was obtained for each of the altitudes of interest.

At the design altitude, the solution converged to a payload ratio of 1.001. This indicates that the effect of lobing is to increase the volume slightly over the design volume. The lobing angle, β, varies continuously along the gore but reaches a local minimum at a point between the maximum radius and the edge of the cap. The maximum amount of lobing occurs at the top of the balloon. The departures from the design values of radius, height and volume are minimal and as a result, all computations must be performed in double precision. The most significant departure is the angle, θ, and the total load near the bottom of the balloon

The meridional and circumferential stress distributions are shown in Figure 6. The results reported by Alexander are also shown in Figure 6 for comparison purposes. It should be noted that the current results have been

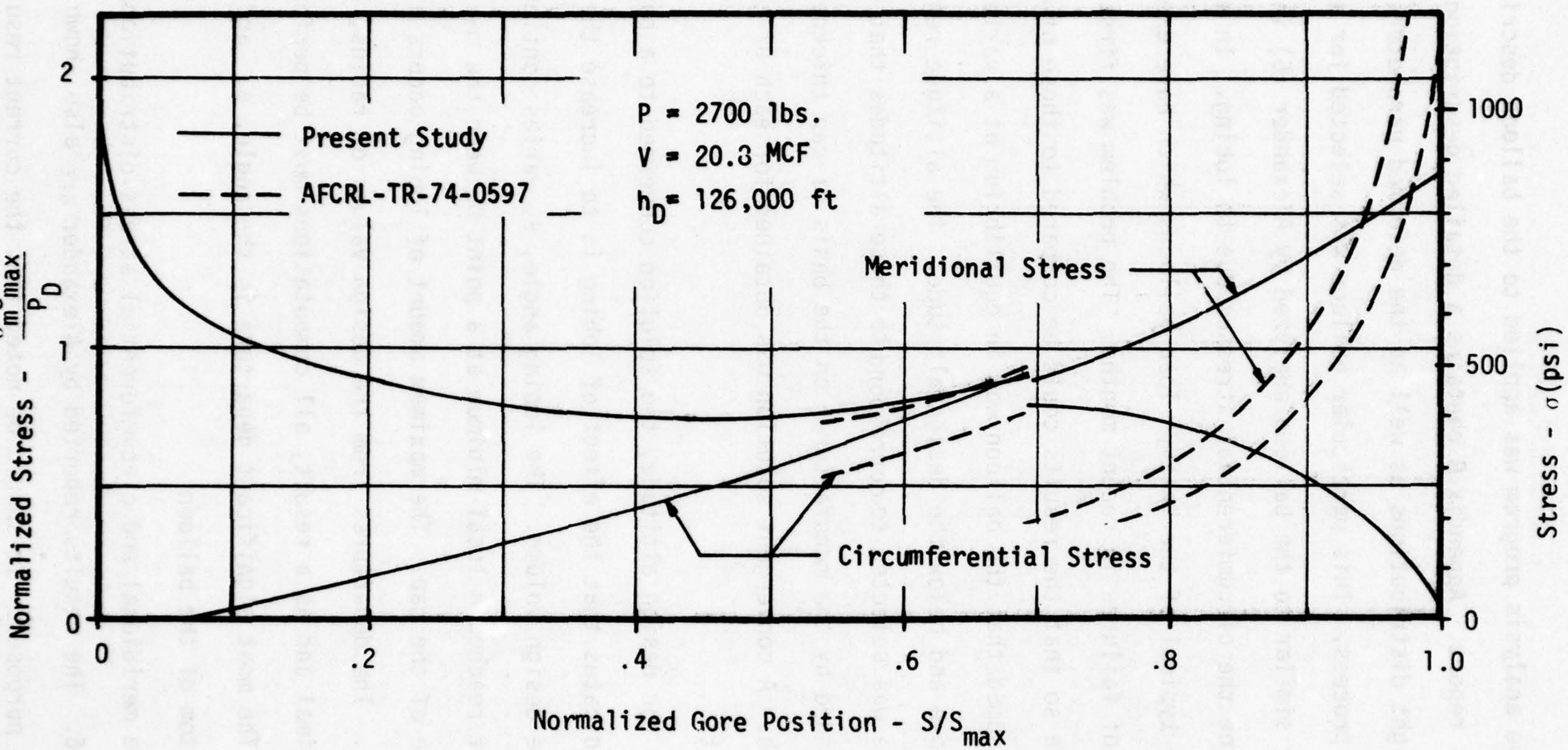


Figure 6. Stress Distributionsat Design Altitude

dimensionalized with respect to the shell thickness. This assumes that the cap is not a load carrying member at the design altitude. It is possible that the large discontinuity in Alexander's results at the edge of the cap is due to not only the weight discontinuity but his assumption that the cap is a load carrying member. However, Alexander's results indicate that the circumferential stress increases as one approaches the apex of the balloon. It is felt that the present results are more reasonable since the circumferential radius of curvature is proportional to the radius, r, and should approach zero as the apex is approached.

The results of the present formulation are presented in Figure 7 for a variety of altitudes of interest. The meridional stresses are shown only for $b/b_D = .8$ and $b/b_D = 1.4$. At any intermediate altitude, the meridional stress will have a value between these two curves. This narrow band is relatively unaffected by changes in altitude of interest. However, the circumferential stress distributions at four different altitudes are also shown in Figure 7. It should be noted that at altitudes below the design altitude the circumferential stresses increase significantly above those predicted at the design altitude. Above the design altitude, in a regime of decreased pressure differential, the circumferential stress is significantly reduced.

Although the stresses predicted by the present analysis are in themselves not considered severe enough to cause failure of this balloon, the region in the shell below the edge of the cap is a region of large biaxial stresses. Any flaw, damage or structural discontinuity in this region could cause a sufficient amplification of these stresses to result in uncontrolled deformation or even failure. This region will be most severe at different times depending on the time of launch. If a morning launch was attempted, the highest magnitude

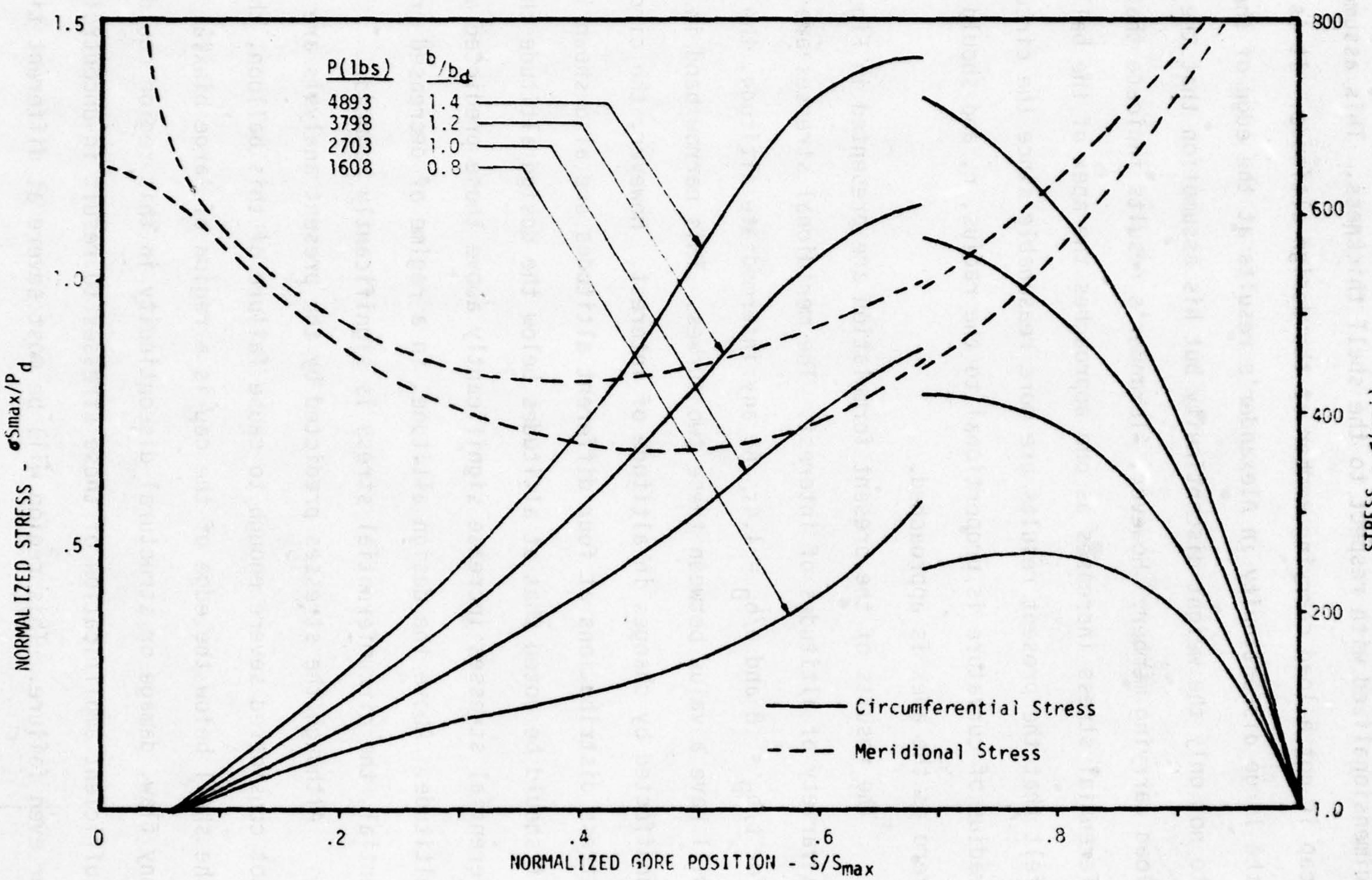


Figure 7 - Stress Distribution in Off Design Configurations

of stress will occur as the balloon with all ballast still on board goes to its equilibrium altitude. As ballast is dropped, the balloon will rise and the stresses will decrease.

Should this balloon be launched in the evening, the system may assume an equilibrium altitude with a subpressure region due to the low gas temperature. However, at sunrise the gas will expand to fill the available volume and the balloon will rise to assume the maximum stress condition.

VIII. CONCLUSIONS

The problem of design and analysis of a single cell balloon has been reformulated in a manner which will permit a more realistic determination of the state of stress in the thin film. The design procedure has been modified to yield a first order estimate of the meridional stress. However, the stresses computed in this manner are not conservative and second order effects must be considered. The effects of lobing have been included in an analysis procedure which produces a realistic distribution of both meridional and circumferential stress. The model formulated in this report suggests a plausible explanation for the transfer of loads between the load tapes and the film.

A computational technique has been developed which is capable of solving the equations formulated in this report. The equations are highly nonlinear and no generalizations may be made nor can the uniqueness of the solution be guaranteed. However, results have been presented for a typical heavy load balloon of interest to many organizations. It has been shown that the circumferential stress is significant, may exceed the meridional stress under certain conditions, and achieves its maximum value at the edge of the cap.

It is hoped that the technique presented in this report will find acceptance by those interested in the successful flight of high altitude balloons. Many observed features such as slack load tapes, stress bands and lobing may now become predictable events.

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APPENDIX A

Balloon Design Program

BALDE

```
C*****************************************************************************
C                          - B A L D E -
C                       BALLOON DESIGN PROGRAM
C*****************************************************************************
C     THIS PROGRAM CALCULATES THE SHAPE OF A FREE BALLOON GIVEN A SET OF
C     INPUT CONDITIONS.  IT IS FOR FULLY TAILORED BALLOONS ONLY, WITH OR
C     WITHOUT AN ENDCAP.  CONVERGENCE OF THE SOLUTION IS OBTAINED BY
C     MATCHING THE TOP LOAD REACTION TO THE TOP LOAD.  THIS PROGRAM
C     INCLUDES THE EFFECTS OF THE LOAD TAPES IN THE CALCULATIONS.
C*****************************************************************************
C     THIS PROGRAM IS BASED ON THE PROGRAMS "BALLOON" AND "FLATBALL",
C     AFCRL-65-92, "DETERMINATION OF THE SHAPE OF A FREE BALLOON", BY
C     J. H. SMALLEY.
C*****************************************************************************
C     DESCRIPTION OF DATA DECK:
C      *  EACH SET OF DATA CONSISTS OF THREE CARDS LISTING THE VALUES
C         IN THE READ STATEMENTS 500 AND 501
C         ALL DATA ARE READ IN 10 WIDE FIELDS OF THE APPROPRIATE TYPE.
C       1 "FIRST" CARD INPUTS ARE AS FOLLOWS:
C            PAYLOAD IN POUNDS
C            ALTITUDE OPTION: 1 = ALT IN FT, 2 = ALT IN MB
C            ALTITUDE IN FEET OR MILLIBARS AS DESIRED
C            FILM TYPE:  1 = POLYETHYLENE, 2 = MYLAR
C            FILM THICKNESS IN INCHES
C            LOAD TAPE TYPE:  1 = POLYESTER, 2 = KEVLAR
C            TAPE LOAD RATING IN POUNDS
C            NUMBER OF LOAD TAPES
C       2 "SECOND" CARD INPUTS ARE AS FOLLOWS:
C            TOP LOAD IN POUNDS, (+) UP, (-) DOWN
C            STRESS CONSTANT TAU0 (USUALLY 0.00)
C            STRESS CONSTANT TAU1 (USUALLY 0.00)
C            SUPERPRESSURE (0.00 FOR NATURAL SHAPE)
C            PRINT INCREMENT N (0 FOR STANDARD OUTPUT)
C            NON-DIMENSIONAL GORE INCREMENT DS0
C            NON-DIMENSIONAL GORE LENGTH TO CAP STARTING LOCATION CSTART
C            FILM THICKNESS INCLUDING CAP IN INCHES
C       3 "THIRD" CARD INPUTS ARE AS FOLLOWS:
C            OUTPUT CONTROL (KEY2=2 FOR PUNCHED DECK OF SHAPE & WEIGHT
C                            KEY2=1 FOR DISK FILE OF SHAPE & WEIGHT)
C            OUTPUT CONTROL (MPT IS NUMBER OF POINTS IN LOAD-ALTITUDE)
C            IDENTIFY LIFTING GAS (FOR HELIUM, IGAS=1)
C            MINIMUM RECOMMENDED PAYLOAD IN POUNDS
C            MAXIMUM RECOMMENDED PAYLOAD IN POUNDS
C      *  IF NO ENDCAP, MAKE SURE THAT CSTART IS GREATER THAN ANY
C         ANTICIPATED GORE LENGTH.
C      *  THE PROGRAM REQUIRES A LAST DATA SET WITH P = 0.0 TO
C         TERMINATE
C*****************************************************************************
C     DESCRIPTION OF OUTPUT:
C         FOR EACH CASE THE INPUT DATA WILL FIRST BE PRINTED, FOLLOWED
C         BY A RECORD OF THE ITERATIONS REQUIRED FOR CONVERGENCE OF THE
C         SOLUTION.  THIS RECORD CONSISTS OF A DISPLAY OF INITIAL AND
C         FINAL ANGLES OF THE GORE WRT THE VERTICAL AXIS OF THE BALLOON.
C         DURING THE FINAL ITERATION, PERTINENT VALUES ALONG THE GORE ARE
C         PRINTED OUT CONTINUOUSLY, FOLLOWED BY A LISTING OF FINAL VALUES.
C*****************************************************************************
C     APPROPRIATE ERROR MESSAGES ARE SUPPLIED IF THE SOLUTION DOES NOT
C     CONVERGE
```

```
******************************************************************
******************************************************************
************************   MAIN PROGRAM   ************************
******************************************************************
      REAL K,KT,KTAPE,LAMEDA,IAM
      INTEGER CCDEF,CCDET
      DIMENSICN GP(100),GW(100),GWT(100),GT(100)
  100 WRITE(6,6C0)
  600 FORMAT('1')
      READ(5,500) P,KEY,CCNST,CCDEF,FTHICK,CODET,TLR,NT
  500 FORMAT(F1C.0,I10,F10.0,I10,F10.4,I10,F10.0,I10)
      READ(5,501) TL,TAU0,TAU1,ALPHA,N,DS0,CSTART,CAP
  501 FCRMAT(F10.2,F10.2,F10.2,F10.2,I10,F10.2,F10.2,F10.4)
C-----------------------------------------------------------------------
C     IF NC MCRE DATA, EXIT.
      IF(P) 2CC,3CC,200
C-----------------------------------------------------------------------
  200 CCNTINUE
      READ(5,502)KEY2,NFT,IGAS,PMIN,PMAX
  502 FCRMAT(3I10,2F10.0)
      CALL DESIGN (P,KEY,CCNST,CCDEF,FTHICK,CODET,TLR,NT,TL,TAU0,TAU1,
     $ALPHA,N,DS0,CSTART,CAP,GP,GW,WBAL,KEY2,S)
C
C             CCMPUTE LCAC - ALTITUDE CURVE
C
      WRITE(6,600)
      WRITE(6,6C1)
  601 FORMAT(50X,'LOAD - ALTITUDE DATA',/42X,'GROSS AIRBORN',/37X,'
     1WEIGHT - (KG)',10X,'ALTITUDE - (KM)')
      GPMIN=1.+WBAL
      GPMAX=GPMIN+(PMAX-PMIN)/P
      DG=(GPMAX-GPMIN)/NFT
      GP1=GPMIN
  250 CCNTINUE
      CALL GLNGTH(GP1,SLMDA,1)
      B1=P/(S/SLMDA)**3/.06243
      CALL BCYNCY(B1,PA,IGAS,TCHIGH)
      CALL ATMOS2(PA,H,TA,RHCA,B,IAM,GXPAN,TOHIGH)
      GRCS=GP1*P*.454
      H=H/1CCC.
      WRITE(6,602)GROS,H
  602 FCRMAT(42X,E15.7,8X,E15.7)
      GP1=GP1+DG
      IF(GP1.GT.GPMAX)GO TO 100
      GC TC 250
  300 CONTINUE
      STCP
      END
```

```
      SUBROUTINE DESIGN (P,KEY,CONST,CODEF,FTHICK,CODET,TLR,NT,TL,TAU0,
     $TAU1,ALPHA,N,DSC,CSTART,CAP,GP,GW,WBAL,KEY2,S)
C*****************************************************************************
C******************        SUBROUTINE DESIGN        *************************
C*****************************************************************************
      REAL K,KT,KTAPE,NU
      REAL LAMBDA
      INTEGER CODEF,CODET
      DIMENSION Y(6),C(6),G(6),E(4),C(4),D(4),WORD1(3),WORD2(3)
      DIMENSION GP(100),GW(100),GWT(100),GT(100)
      DIMENSION SSTORE(300),GSTORE(300),WSTORE(300),TSTORE(300)
      REAL F1(3)/'POLY','ETHY','LENE'/
      REAL F2(3)/'MYLA','R   ','    '/
      REAL T1(3)/'POLY','ESTE','R   '/
      REAL T2(3)/'KEVL','AR  ','    '/
      REAL K1/'  FT'/
      REAL K2/'  MB'/
C
C     B, C, AND D ARE CONSTANTS USED IN GILL'S MODIFIED R-K METHOD FOR
C     SOLVING THE GOVERNING DIFFERENTIAL EQUATIONS.
      DATA B/2.0,1.0,1.0,2.0/, C/0.5,0.29289,1.7071,0.5/, D/0.5,
     $0.29289,1.7071,0.16667/
C
      CALL MATL(P,KEY,CONST,CODEF,FTHICK,CODET,TLR,NT,CAP,SIGMA,CSIGMA,
     $TSIGMA,KT,EM,EC,EMC,LAMBDA,TSIGME)
      NU=EMC/EM
      OMEGA1=SIGMA/1.8453
      OMEGA2=OMEGA1*(CSIGMA/SIGMA)
      OMEGA=OMEGA1+TSIGMA/1.8453
      TOMEGA=TSIGMA/6.2832
C
C     EMPIRICAL CURVE FITS FROM SMALLEY'S REPORT, TO GET GOOD FIRST ESTIMAT
C     OF THETA0.  GOOD FOR SIGMA BETWEEN 0.0 AND 0.8 .
      THETA0=1.347
      IF(OMEGA.LT.0.43354) THETA0=0.72*OMEGA+1.1015
      IF(OMEGA.LT.0.35225) THETA0=1.15*OMEGA+0.9507
      IF(OMEGA.LT.0.27096) THETA0=1.46*OMEGA+0.8687
      OMEGA=OMEGA1
C
      IQUIT = 0
      LSTRUN=1
      IF(CODEF.EQ.2) GO TO 52
      DO 51   I=1,3
   51 WORD1(I)=F1(I)
      GO TO 54
   52 DO 53   I=1,3
   53 WORD1(I)=F2(I)
   54 IF(CODET.EQ.2) GO TO 56
      DO 55   I=1,3
   55 WORD2(I)=T1(I)
      GO TO 58
   56 DO 57   I=1,3
   57 WORD2(I)=T2(I)
   58 IF(KEY.EQ.2) GO TO 59
      WORD3=K1
      GO TO 60
   59 WORD3=K2
   60 CONTINUE
```

```
      WRITE(6,601) (WORD1(I),I=1,3),FTHICK,SIGMA,TAU0,(WORD2(I),I=1,3),
     $EM,TAU1,TLR,EC,ALPHA
  601 FORMAT(5X,37('*')/5X,'*',35X,'*',15X,'NON-DIMENSIONAL QUANTITIES',
     18X,'MISC. PARAMETERS'/5X,'*  FILM TYPE',8X,': ',A4,A4,A4,'  *'/5X,
     2'*  FILM THICKNESS    : ',F7.4,' IN',4X,'*',17X,'SIGMA FILM =',F9.4
     3,12X,'TAUC  =',F6.3/5X,'*  TAPE TYPE',8X,': ',A4,A4,A4,'  *',17X,
     4'EM',9X,'=',F8.3,13X,'TAU1    =',F6.3/5X,'*  TAPE LOAD RATING : ',
     5F6.0,'  LBS    *',17X,'EC',9X,'=',F8.3,13X,'ALPHA =',F6.3)
      WRITE(6,602) NT,EMC,DSC,P,TSIGMA,CSTART,TL,KT,N,CONST,WORD3
  602 FORMAT(' ',4X,'*  NUMBER OF TAPES    : ',I5, 9X,'*',17X,'EMC',8X,
     1'=',F8.3,13X,'DSO    =',F6.3/5X,'*   PAYLOAD',10X,': ',F6.0,'  LBS
     2 *',17X,'SIGMA TAPE =',F9.4,12X,'CSTART=',F5.2/5X,'*   (INCL TOPLOA
     3D CF): ',F6.0,'  LBS    *',17X,'KT',9X,'=',F6.1,15X,'N     =',I2/5X
     4,'*   DESIGN ALTITUDE   : ',F7.0,A4,'    *'/5X,'*',35X,'*'/5X,37('*')
     5,///5X, 'ITERATION RECORD (INITIAL AND FINAL ANGLES)'//)
C-----------------------------------------------------------------------
C     IF THE SOLUTION IS NOT CONVERGING, EXIT
   21 IQUIT = IQUIT+1
      IF(IQUIT.LT.20) GO TO 22
      WRITE(6,603)
  603 FORMAT('-',' *** SOLUTION IS NOT CONVERGING - MAXIMUM NUMBER OF IT
     $ERATIONS HAS BEEN EXCEEDED *** ')
      GO TO 5
C
C     LSTRUN = 0 IS THE KEY FOR LAST ITERATION.  IT IS AN OUTPUT CONTROL.
   22 IF(LSTRUN.NE.0)GO TO 23
C-----------------------------------------------------------------------
      WRITE(6,604)
  604 FORMAT('0',95X,27HMANUFACTURER'S TABLE LAYOUT//10X,'S',13X,'R',
     113X,'Z',13X,'T',11X,'TAUM',9X,'TAUC',8X,'GORE POSITION (FT)',5X,
     2'HALF GORE WIDTH (IN)'/)
   23 DO 24 I=1,6
   24 Q(I)=0.
  231 DO 241 I=1,300
      SSTORE(I)=0.0
      WSTORE(I)=0.
      TSTORE(I)=0.
  241 GSTORE(I)=0.0
      JCOUNT=1
      DS = DSO
      SSTOP = CSTART
      OMEGA=OMEGA1
      TOMEGA=TSIGMA/6.2832
      TW=0.
      ITAG=0
      RC=0.
      GW2=0.
      S0=0.
      S0DIM=0.
      R=0.
      S=0.
      Z=0.
      WSUM=0.
      ASUM=0.
      TAUM=0.
      TAUC=TAU0
      THETA=THETAC
      RT0=1./6.2832/COS(THETA)
```

```
      Y(1)=THETA
      Y(2)=R
      Y(3)=Z+ALPHA
      Y(4)=((F+TL)/P)/6.2832/COS(THETA)
      Y(5)=C.C
      Y(6)=C.C
      SSTORE(JCCUNT) = SODIM
      GSTORE(JCCUNT) = GW2
    2 T=6.2832*Y(4)
      IF(LSTRUN)34,32,34
   32 WRITE(6,605) S,R,Z,T,TAUM,TAUC,SODIM,GW2
  605 FORMAT(4(4X,F10.5),4X,F8.3,5X,F8.3,12X,F7.1,17X,F7.2)
      IF(KEY2.EQ.1)WRITE(3,650)Y(2),Y(3),Y(4),Y(1),Y(6)
      IF(KEY2.EQ.2)WRITE(7,650)Y(2),Y(3),Y(4),Y(1),Y(6)
  650 FORMAT(5E15.7)
C-----------------------------------------------------------------------
C     ICCUNT AND N ARE OUTPUT CONTROLS USED DURING THE LAST ITERATION.
C     N IS THE NUMBER OF INCREMENTS THAT WILL BE SKIPPED IN THE PRINTOUT.
C
   34 ICCUNT=N
C-----------------------------------------------------------------------
C     ITAG SENDS THE PROGRAM INTO ITERATIONS AND RELEASES IT WHEN THE TOP
C     OF THE BALLCON HAS BEEN REACHED
      IF(ITAG-1)4,3,4
C-----------------------------------------------------------------------
    3 IF(LSTRUN.EQ.0)GO TO 35
      DEGO = THETAO*57.2956
      THETAD = THETA*57.2956
      DS = DSC
      WRITE(6,611) DEGO,THETAD
  611 FORMAT(' ',5X,F10.5,5X,F10.5)
C-----------------------------------------------------------------------
C     CONVERGENCE TEST: TOP LOAD REACTION WITHIN .5 LB OF APPLIED TOP LOAD.
      F = -6.2832*Y(4)*COS(THETA)*P
      IF(ABS(F+TL).LE.0.50) LSTRUN=0
C
C     COMPUTE CORRECTIVE TERM ON THETAO
C     DELTA(THETAO)/DELTA(THETA) IS GENERALLY APPROX.= (+1/-2)
C     DTHETA IS THE DESIRED VALUE OF THETA TO MATCH THE TOP LOAD REACTION
C     TO THE APPLIED TOP LOAD.
      DTHETA = -(ARCOS(TL/(6.2832*P*Y(4))))
      DELTA = DTHETA - THETA
      CORR = -DELTA/2.
      THETAO = THETAO+CORR
C
      GO TO 21
C-----------------------------------------------------------------------
C
C     COMPUTE AND PRINT FINAL QUANTITIES
   35 VOL = 3.1416*Y(6)*(LAMEDA**3)
      ASUM = (ASUM+6.2832*Y(5))*(LAMBDA**2)
      WSUM = (WSUM+6.2832*OMEGA*Y(5))*P
      TW=TW*6.2832*P
      WEIGHT = WSUM+TW
      F = -6.2832*Y(4)*COS(THETA)*P
      THETAD=57.2956*THETA
      DEGO = THETAO*57.2956
      JMAX = JCCUNT
```

```
      PTOP=-TL/P
      WBAL=WEIGHT/P
      WRITE(6,606) LAMBDA,VOL,DEG0,ASUM,THETAD,WSUM,F,TW,WEIGHT
  606 FORMAT('-',4X,'FINAL QUANTITIES (LAMBDA =',F7.1,' FT): '//5X,'VOLUM
     1E',6X,'=',E10.3,' FT3',10X,'INITIAL ANGLE',5X,'=',F7.3,' DEG'/5X,
     2'AREA',8X,'=',E10.3,' FT2',10X,'FINAL ANGLE',7X,'=',F7.3,' DEG'
     3/5X,'FILM WEIGHT =',F7.0,'      LBS',10X,'TCP LOAD REACTION =',F7.3,
     4' LBS'/5X,'TAPE WEIGHT =',F7.0,'      LBS'/5X,'TOT. WEIGHT =',F7.0,
     5'      LBS'/)
      IF(KEY2.EQ.1)WRITE(3,650)P,PTOP,WBAL,S,LAMBDA
      IF(KEY2.EQ.2)WRITE(7,650)P,PTOP,WBAL,S,LAMBDA
      IF(KEY2.EQ.1)WRITE(3,651)NU,KT,EM,NT
      IF(KEY2.EQ.2)WRITE(7,651)NU,KT,EM,NT
  651 FORMAT(3E15.7,I15)
C
C     GENERATE TRANSFER MATRICES ... 100 EVENLY SPACED, UNDEFORMED
C     GORE COORDINATES
C
      DSNEW = SSTORE(JMAX)/SS.
      J=1
      SNEW=0.0
      GP(J)=0.0
      GW(J)=0.0
      GWT(J)=TOMEGA
      GT(J)=TOMEGA*6.2832/NT
      WRITE(6,600) J,GP(J),J,GW(J),J,GWT(J),J,GT(J)
  100 SNEW = SNEW+DSNEW
      J = J+1
      DO 101 I=1,JMAX
      QUAN = SNEW-SSTORE(I)
      IF(QUAN.LT.0.0) GO TO 102
  101 CONTINUE
  102 GP(J) = SNEW
      GW(J) = GSTORE(I-1)+((SNEW-SSTORE(I-1))*(GSTORE(I)-GSTORE(I-1))/
     $(SSTORE(I)-SSTORE(I-1)))
      GWT(J)= WSTORE(I-1)+((SNEW-SSTORE(I-1))*(WSTORE(I)-WSTORE(I-1))/
     $(SSTORE(I)-SSTORE(I-1)))
      GT(J) = TSTORE(I-1)+((SNEW-SSTORE(I-1))*(TSTORE(I)-TSTORE(I-1))/
     $(SSTORE(I)-SSTORE(I-1)))
      WRITE(6,600) J,GP(J),J,GW(J),J,GWT(J),J,GT(J)
  600 FORMAT(' ',5X,'GP(',I3,') = ',F10.3,10X,'GW(',I3,') =',F10.3,10X,
     1'GWT(',I3,') = ',F10.3,10X,'GT(',I3,') = ',F10.3)
  652 FORMAT(4E15.7)
      IF(J.EQ.100) GO TO 103
      GO TO 100
  103 CONTINUE
      DO 300 J=1,JMAX
      IF(KEY2.EQ.1)WRITE(3,652)SSTORE(J),GSTORE(J),WSTORE(J),TSTORE(J)
      IF(KEY2.EQ.2)WRITE(7,652)SSTORE(J),GSTORE(J),WSTORE(J),TSTORE(J)
  300 CONTINUE
C
      GO TO 5
C-----------------------------------------------------------------------
C     COMPUTATIONAL PORTION
C-----------------------------------------------------------------------
C
C     CHECK IF OVER THE SHOULDER OF THE BALLOON
    4 IF(SIN(THETA))6,8,8
```

```
C     IF SO, CHECK IF WITHIN DS OF THE TOP
    6 IF(R-DS)7,7,8
C     IF SO, REDEFINE DS, KEY WITH ITAG
    7 DS=R/ABS(SIN(THETA))
      ITAG=1
C--------------------------------------------------------------------
C     PROGRAM SOLVING STARTS HERE
C
C     SET UP FOR GILL'S METHOD
    8 DO 10 J=1,4
      G(2)=SIN(Y(1))
      G(3)=COS(Y(1))
      G(1)=(TAUC*G(3)-Y(2)*Y(3)-G(2)*Y(2)*OMEGA-TOMEGA*G(2))/Y(4)
      G(4)=TAUC*G(2)+G(3)*(OMEGA*Y(2)+TOMEGA)
      G(5)=Y(2)
      G(6)=Y(2)*Y(2)*G(3)
C
C     GILL'S METHOD
      DO 9 I=1,6
      P1 = D(J)*(G(I)-B(J)*Q(I))
      Y(I) = Y(I)+DS*P1
    9 Q(I) = Q(I)+3.0*P1-C(J)*G(I)
C
   10 CONTINUE
C--------------------------------------------------------------------
      TW=TW+TOMEGA*DS
      THETA=Y(1)
      R=Y(2)
      Z=Y(3)-ALPHA
      S=S+DS
      JCOUNT = JCOUNT+1
      IF(R) 161,161,16
  161 R=1.E-10
   16 TAUC=TAU0+(TAU1*(Y(4)-RT0))/R
      EPSM=(Y(4)/R-EMC*TAUC/EC)/(EM-EMC*EMC/EC+KT/R/6.2832)
      EPST=EPSM
      TAUM=(Y(4)-KT*EPST/6.2832)/R
      R0=R/(1.+(TAUC-(EMC*EPSM)/EC))
      GW2=3.1416*R0*LAMBDA*12./NT
      S0=S0+(DS/(1.+EPSM))
      S0DIM=S0*LAMBDA
      SSTORE(JCOUNT) = S0DIM
      GSTORE(JCOUNT) = GW2
      WSTORE(JCOUNT)=Y(2)*OMEGA+TOMEGA
      TSTORE(JCOUNT)=TOMEGA*6.2832/NT
C--------------------------------------------------------------------
C     IF TAUC < 0, BUCKLING OCCURS, SO SET VALUES TO 0
C
      IF(TAUC)31,17,17
   31 TAUC=0.
      TAU0=0.0
      TAU1=0.0
C--------------------------------------------------------------------
C     CHECKS FOR IMPOSSIBLE BALLOONS.
C     IF |THETA|>PI OR S>10.0, PRINT AN APPROPRIATE ERROR MESSAGE AND
C     PROCEED TO THE NEXT SET OF DATA
```

```
   17 IF((ABS(THETA)-3.1416).LT.0.0) GC TC 12
      WRITE(6,607)
  607 FORMAT('-',' *** SOLUTICN IS NCT CONVERGING - IMPOSSIBLE BALLOON :
     $ |THETA| > PI *** ')
      GC TC 5
   12 IF((S-10.0).LT.0.0) GC TC 30
      WRITE(6,608)
  608 FORMAT('-',' *** SOLUTICN IS NOT CONVERGING - IMPOSSIBLE BALLOON :
     $ NON-DIM S > 10.0 *** ')
      GO TO 5
C-----------------------------------------------------------------------
C     THIS PORTION ALLOWS FOR THE CHANGE IN FILM THICKNESS DUE TO THE CAP
C
   30 IF(S-SSTCP)14,13,13
   13 T=6.2832*Y(4)
  131 ASUM=ASUM+6.2832*Y(5)
      WSUM=WSUM+6.2832*CMEGA*Y(5)
      Y(5)=0.
      IF(LSTRUN.NE.0) GC TC 132
  609 FORMAT('0',3X,'CAP STARTS AT S =',F6.2,  ' ... SIGMA CAP =',F7.4/)
      WRITE(6,609) CSTART,CSIGMA
  132 OMEGA = CMEGA2
      TOMEGA=TSIGMB/6.2832
      SSTCP = 10.0
   14 IF(ICOUNT)2,2,15
   15 ICOUNT=ICOUNT-1
      IF(ITAG)2,4,2
    5 CONTINUE
      S=S*LAMBDA
      RETURN
      END
```

```
      SUBROUTINE MATL(P,KEY,CONST,CODEF,FTHICK,CODET,TLR,NT,CAP,SIGMA,
     $CSIGMA,TSIGMA,KT,EM,EC,EMC,LAMEDA,TSIGMB)
C***********************************************************************
C     THIS SUBROUTINE CALCULATES MATERIAL PROPERTIES GIVEN INFORMATION
C     AS FOLLOWS:
C         INPUTS:  PAYLOAD, ALTITUDE OPTION KEY, ALTITUDE, FILM TYPE AND
C                THICKNESS, TAPE TYPE AND LOAD RATING, NUMBER OF TAPES, AND
C                CAP THICKNESS
C         OUTPUTS:  NON-DIMENSIONAL FILM AND TAPE WEIGHTS, TAPE STIFF-
C                NESS, AND FILM MODULI.
C***********************************************************************
C     ALL EMPIRICAL RELATIONSHIPS USED ARE SUBJECT TO REVISION WHEN
C     ADDITIONAL TEST DATA BECOME AVAILABLE.
C***********************************************************************
C     THIS SUBROUTINE ASSUMES, FOR LACK OF A GOOD TEMPERATURE MODEL, THAT
C     THE BALLOON MATERIAL TEMPERATURE IS ATMOSPHERIC TEMPERATURE.
C     THIS ASSUMPTION MAY NOT BE ADEQUATE.
C***********************************************************************
      REAL K,KT,KTAPE,LAMBDA
      REAL MG,MA,LF,IAM
      INTEGER CODEF,CODET
C-----------------------------------------------------------------------
C     100 SERIES - DEFINE NECESSARY VALUES
C-----------------------------------------------------------------------
      IF(KEY.EQ.2) GO TO 101
  100 H=CONST
C
      H=H*0.3048
C     CONVERSION OF H FROM FEET TO METERS FOR SUBRT ATMOS
C
      CALL ATMOS(H,TA,PA,RHOA,B,IAM,GXPAN,TOHIGH)
      GO TO 102
  101 PA=CONST
      CALL ATMOS2(PA,H,TA,RHOA,B,IAM,GXPAN,TOHIGH)
C
  102 B=B*6.243E-02
C     CONVERSION OF B FROM KG/M3 TO LBM/FT3
C
      T=TA-273.16
C     CONVERSION OF TEMPERATURE FROM DEG(R) TO DEG(C)
C
      LAMBDA = (P/B)**(1./3.)
      K = 1./((6.283185)**(1./3.))
C-----------------------------------------------------------------------
C     200 SERIES - COMPUTE FILM PROPERTIES
C-----------------------------------------------------------------------
      IF(CODEF.EQ.2) GO TO 202
  201 WFILM = 58.06*(FTHICK/12.)
      EFILM = (0.02*(T**2)-1.60*(T)+41.95)*1000.
      GO TO 203
  202 WFILM = 87.08*(FTHICK/12.)
C
C     EFILM = ?????      MYLAR FILM DATA IS NECESSARY
C
  203 SIGMA = WFILM/(K*B*LAMBDA)
      CSIGMA=SIGMA*(CAP/FTHICK)
C
C     THIS PROGRAM ASSUMES A POISSON'S RATIO OF 0.50
```

```
C
      EM = (EFILM*FTHICK*12.)/(B*(LAMBDA**2))
      EC = EM
      EMC = EM/2.
C-------------------------------------------------------------------------------
C     300 SERIES - COMPUTE TAPE PROPERTIES
C-------------------------------------------------------------------------------
      IF(CODET.EQ.2) GO TO 302
  301 WTAPE = (6.1E-06)*TLR+0.0038+.3024*(.003+FTHICK)
      WTAPE2=WTAPE+.3024*(CAF-FTHICK)
      KTAPE = (TLR*10.)*(1.-(6.67E-03)*(T-20.))
      GO TO 303
  302 WTAPE = (2.0E-06)*TLR+0.0021
      KTAPE = 26.*TLR
C
C     NO INFORMATION ON LOW TEMPERATURE EFFECTS CURRENTLY AVAILABLE
C
  303 TSIGMA = (NT*WTAPE)/(B*(LAMBDA**2))
      TSIGMB=TSIGMA*WTAPE2/WTAPE
      KT = (NT*KTAPE)/(B*(LAMBDA**3))
      RETURN
      END
```

```
      SUBROUTINE ATMOS(H,TA,PA,RHOA,B,IAM,GXPAN,TOHIGH)
C
C ...... SUBROUTINE FOR SOLVING FOR THE VALUES OF TEMPERATURE,
C ...... PRESSURE, DENSITY, SPECIFIC BOUYANCY, INTEGRATED AIR
C ...... MASS, AND "GAS EXPANSION" FOR ANY GIVEN ALTITUDE BELOW
C ...... 61000. METERS (200131. FEET). ALL EQUATIONS HAVE BEEN
C ...... DERIVED ACCORDING TO THE U.S. STANDARD ATMOSPHERE, 1962.
C ...... --- ALL UNITS ARE IN THE SI SYSTEM, I.E.
C ...... TEMPERATURE(DEG KELVIN), PRESSURE(MB), DENSITY(KG/M3),
C ...... SPECIFIC BOUYANCY(KG/M3) INTEGRATED AIR MASS(KG/M2),
C ...... GAS EXPANSION(DIMENSIONLESS).--- LIFTS WERE BASED ON
C ...... GRADE A HELIUM AND PURE HYDROGEN.
C
C       FOR SPECIFIC BOUYANCY, TO CONVERT FROM KG/M3 TO LB/FT3,
C       MULTIPLY BY 0.06243
C
C       H IS INPUT IN METERS
C
      REAL MG,MA,LP,IAM
      TOHIGH =1
      MG=4.0026
      MA=28.9644
      R=8.31432E03
      GY=9.80665
      RHOSTP=1.2250
      IGAS=1
C     IF HYDROGEN GAS IS USED INSTEAD OF HELIUM,IGAS MUST BE CHANGED TO
C     SOMETHING OTHER THAN 1
C
      IF(H.LT.61000.0) GO TO 10
      WRITE(6,155) H
  155 FORMAT('0',5X,'***** ALTITUDE H OUTSIDE RANGE OF SUBROUTINE ATMOS.
     1  H =',E14.7,' METERS. ******')
      TOHIGH=-1.
      GO TO 28
   10 IF (H.LT.52000.0)GO TO 12
      HB=52000.0
      TB=270.65
      LP=-0.0020
      PB=0.590
      RHOB=0.0007594
      GO TO 22
   12 IF(H.LT.47000.0) GO TO 14
      HB=47000.0
      TB=270.65
      LP=0.0
      PB=1.109
      RHOB=0.0014275
      GO TO 22
   14 IF(H.LT.32000.0) GO TO 16
      HB=32000.0
      TB=228.65
      LP=0.0028
      PB=8.680
      RHOB=0.013225
      GO TO 22
   16 IF(H.LT. 20000.0 )GO TO 18
      HB=20000.0
```

```
      TB=216.65
      LP=0.0010
      PB=54.749
      RHOB=0.088035
      GO TO 22
   18 IF (H.LT.11000.0)GO TO 20
      HB=11000.0
      TB=216.65
      LP=0.0
      PB=226.320
      RHOB=0.36392
      GO TO 22
   20 HB=0.0
      TB=288.15
      LP=-0.0065
      PB=1013.250
      RHOB=1.2250
   22 TA=TB+LP*(H-HB)
      IF(LP.EQ.0)GO TO 24
      PA=PB/(LP*(H-HB)/TB+1.0)**(GY*MA/LP/R)
      GO TO 26
   24 PA=PB/EXP(GY*MA/R/TB*(H-HB))
   26 RHOA= PA*MA/R/TA*100.0
      IF(IGAS.NE.1) MG=2.0159
      B=RHOA*(1.0-MG/MA)
      IAM=10.0*PA*(1.03751-0.00527*ALOG10(PA))
      GXPAN=RHOSTP/RHOA
   28 RETURN
      END
```

```
      SUBRCUTINE ATMCS2(PA,H,TA,RHOA,B, IAM,GXFAN,TOHIGH)
C
C ....... SUEROUTINE FOR SCLVING FOR THE VALUES OF TEMPERATURE,
C ....... ALTITUDE, DENSITY, SPECIFIC BCUYANCY, INTEGRATED AIR
C ....... MASS, AND "GAS EXPANSION" FOR ANY GIVEN ALTITUDE BELOW
C ....... 61000. METERS (200131. FEET). ALL EQUATIONS HAVE BEEN
C ....... DERIVED ACCORDING TO THE U.S. STANDARD ATMOSPHERE, 1962.
C ....... --- ALL UNITS ARE IN THE SI SYSTEM, I.E.
C ....... TEMPERATURE(DEG KELVIN), PRESSURE(MB), DENSITY(KG/M3),
C ....... SPECIFIC BOUYANCY(KG/M3) INTEGRATED AIR MASS(KG/M2),
C ....... GAS EXPANSICN(DIMENSICNLESS).--- LIFTS WERE BASED ON
C ....... GRADE A HELIUM AND PURE HYDROGEN.
C
C         FOR SPECIFIC BCUYANCY, TO CONVERT FROM KG/M3 TO LB/FT3,
C         MULTIPLY BY 0.06243
C
C         PA IS INPUT IN MB
C
      REAL MG,MA,LP,IAM
      TOHIGH=1
      MG=4.0026
      MA=28.9664
      R=8.31432E03
      GY=9.80665
      RHOSTP=1.2250
      IGAS=1
      IF(PA.LT.226.32) GC TC 10
      PB=1013.25
      HB=0.0
      TB=288.15
      LP=-0.0065
      RHOB=1.225
      GC TO 22
   10 IF(PA.LT.54.749) GO TO 12
      PB=226.32
      HB=11000.
      TB=216.65
      LP=0.0
      RHOB=0.36392
      GC TO 22
   12 IF(PA.LT.8.680) GC TC 14
      PB=54.749
      HB=20000.
      TB=216.65
      LP=0.001
      RHOB=0.088035
      GO TO 22
   14 IF(PA.LT.1.109) GO TO 16
      PB=8.680
      HB=32000.
      TB=228.65
      LP=0.0028
      RHOB=0.013225
      GC TO 22
   16 IF(PA.LT.0.590) GC TO 18
      PB=1.109
      HB=47000.
      TB=270.65
```

```
      LP=0.0
      RHOB=0.0014275
      GO TO 22
 18   IF(PA.LT.0.1828) GO TO 20
      PB=0.59
      HB=52000.
      TB=270.65
      LP=-0.002
      RHOB=0.0007594
      GO TO 22
 20   WRITE(6,155) PA
      WRITE(6,155) PA
155   FORMAT('0',5X,'***** ATMOSPHERIC PRESSURE PA OUTSIDE RANGE OF SUBR
     1OUTINE ATMOS2.   PA =',E14.7,' MB. *****')
      TCHIGH=-1.
      GO TO 28
 22   IF(LP.EQ.0.0) GO TO 24
      H=HB+TB/LP*((PB/PA)**(LP*R/GY/MA)-1.0)
      GO TO 26
 24   H=ALOG(PB/PA)*R*TB/GY/MA+HB
 26   TA=TB+LP*(H-HB)
      RHOA=PA*MA/R/TA*100.0
      IF(IGAS.NE.1) MG=2.0159
      B=RHOA*(1.0-MG/MA)
      IAM=10.0*PA*(1.03751-0.00527*ALOG10(PA))
      GXFAN=RHOSTP/RHOA
 28   RETURN
      END
```

```
      SUBROUTINE BOYNCY(B,PA,IGAS,TOHIGH)
C
C
C ...... SUBROUTINE TO SOLVE FOR ATMOSPHERIC PRESSURE (PA) GIVEN THE
C ...... CORRESPONDING GAS BOUYANCY (B) AT ANY ALTITUDE BELOW 61,000
C ...... METERS (200,131 FEET).  ALL EQUATIONS HAVE BEEN DERIVED ACCORDING
C ...... TO THE U.S. STANDARD ATMOSPHERE, 1962.
C ...... ----- ALL UNITS ARE IN THE SI SYSTEM:
C ......
C ...... SPECIFIC BOUYANCY (B) IS THE INPUT IN KG/M3
C ...... ATMOSPHERIC PRESSURE (PA) IS THE OUTPUT IN MB
C ......
C ......CALCULATIONS ARE BASED ON PURE HYDROGEN AND GRADE A HELIUM.
C ...... FOR HYDROGEN, INPUT IGAS AS 0.
C ...... FOR HELIUM, INPUT IGAS AS 1.
C ......
C ...... TOHIGH WILL BE SET TO  -1  IF BOUYANCY IS OUTSIDE THE PARAMETERS
C ...... OF THE STANDARD ATMOSPHERE.
C
C
C
      REAL MW
      TOHIGH=1
      MW=4.0026
      B1=1.0557
      B2=0.31355
      B3=0.075858
      B4=0.011396
      B5=1.2308E-3
      B6=6.5444E-4
      B7=2.3635E-4
      IF(IGAS.EQ.1)GO TO 90
      MW=2.0159
      B1=1.1397
      B2=0.33853
      B3=0.081899
      B4=0.012303
      B5=1.3288E-3
      B6=7.0656E-4
      B7=2.5517E-4
   90 IF(B.LT.B1)GO TO 1
      WRITE(6,101) B
  101 FORMAT('0','********** VALUE OF B IS OUTSIDE LIMITS OF STANDARD AT
     1MOSPHERE.  B = ',E12.5,' (KG/M3) **********')
      TOHIGH=-1.
      GO TO 40
    1 IF(B.LE.B2)GO TO 2
      TB=288.15
      XLP=-0.0065
      PB=1013.250
      GO TO 9
    2 IF(B.LE.B3)GO TO 3
      TB=216.65
      XLP=0.0
      PB=226.320
      GO TO 9
    3 IF(B.LE.B4)GO TO 4
      TB=216.65
```

```
      XLP=0.0010
      PB=54.749
      GO TO 9
    4 IF(B.LE.B5)GO TO 5
      TB=228.65
      XLP=0.0028
      PB=8.680
      GO TO 9
    5 IF(B.LE.B6)GO TO 6
      TB=270.65
      XLP=0.0
      PB=1.109
      GO TO 9
    6 IF(B.LE.B7)GO TO 7
      TB=270.65
      XLP=-0.0020
      PB=0.590
      GO TO 9
    7 WRITE(6,101) B
      TCHIGH=-1.
      GO TO 40
    9 ITTER=0
      PNEW=PB
   10 POLD=PNEW
      ITTER=ITTER+1
      FPA=POLD-(2.870531*B*TB/(1.-MW/28.9644))*(1.+((PB/POLD)**(29.2713*
     1XLP)-1.))
      GPA=1.+(84.0241*B*TB*XLP*PB/((1.-MW/28.9644)*POLD**2))*(PB/POLD)**
     1(29.2713*XLP-1.)
      PNEW=POLD-FPA/GPA
      IF(ABS((PNEW-POLD)/PNEW).LE..00001)GO TO 20
      IF(ITTER.GE.50)GO TO 30
      GO TO 10
   30 DELPA=PNEW-POLD
      WRITE(6,100) DELPA
  100 FORMAT('0',10X,'SOLUTION DID NOT CONVERGE AFTER 50 ITTERATIONS.'
     1/5X,'DIFFERENCE BETWEEN NEW AND OLD PRESSURE ON LAST ITTERATION WA
     2S',2X,E10.3)
      TCHIGH=-1.
   20 PA=PNEW
   40 RETURN
      END
```

```
      SUBROUTINE GLNGTH(GP,SLMDA,INDIC)
C
C
C ...... SUBRCUTINE TC SOLVE FOR THE GORE LENGTH RELATICNS OF ZERO-PRESSUR
C ...... NATURAL-SHAPE, FLAT-TOP BALLOONS.  TO OBTAIN GORE LENGTH/LAMBDA
C ...... (SLMDA) FRCM GRCSS/PAYLCAD (GP) INITIALIZE INDIC TO 1.  TO OBTAIN
C ...... THE REVERSE OF THE ABOVE, INTIALIZE INDIC TO ZERO.
C ......
C ...... THE FOLLOWING IS A CURVE-FIT CF FIGURE 5 IN NCAR-TN/IA-99, SECTIO
C
      IF(INDIC.EQ.0)GO TO 60
      IF(GP.GE.1.)GC TC 10
   40 WRITE(6,100) GP
  100 FORMAT('0','********** VALUE OF GROSS/PAYLOAD OUTSIDE RANGE OF SUB
     1ROUTINE GLNGTH.  GROSS/PAYLOAD(GP) = ',E12.5,'**********')
      INDIC=-1
      GO TO 99
   10 IF(GP.GE.1.1458)GO TO 11
      A=1.99442
      B=0.274849
      GO TO 50
   11 IF(GP.GE.1.3205)GC TC 12
      A=1.99287
      B=0.280545
      GC TO 50
   12 IF(GP.GE.1.5306)GO TO 13
      A=1.989503
      B=0.286635
      GO TO 50
   13 IF(GP.GE.1.7838)GC TO 14
      A=1.98507
      B=0.291881
      GO TO 5C
   14 IF(GP.GE.2.0897)GO TO 15
      A=1.97853
      B=C.297581
      GC TC 50
   15 IF(GP.GE.2.4586)GC TC 16
      A=1.97175
      B=0.302239
      GC TO 5C
   16 IF(GP.GE.2.9035)GO TO 17
      A=1.96328
      B=0.307023
      GO TO 50
   17 IF(GP.GE.3.4380)GC TC 18
      A=1.95482
      B=0.311076
      GC TO 50
   18 IF(GP.GE.4.0778)GO TO 19
      A=1.94521
      B=C.315067
      GO TO 50
   19 IF(GP.GE.4.8380)GC TC 20
      A=1.93701
      B=0.318071
      GC TO 50
   20 IF(GP.GE.5.7363)GO TO 21
```

```
      A=1.92864
      B=0.320819
      GO TO 50
   21 IF(GP.GE.6.7898)GO TO 22
      A=1.92034
      B=0.323288
      GO TO 50
   22 IF(GP.GE.8.0155)GO TO 23
      A=1.91325
      B=0.325220
      GO TO 50
   23 IF(GP.GT.10.)GO TO 40
      A=1.90667
      B=0.326875
   50 SLMDA=A*GP**B
      GO TO 99
   60 IF(SLMDA.GE.1.99442)GO TO 70
   91 WRITE(6,200) SLMDA
  200 FORMAT('0','*********** VALUE OF GORE LENGTH/LAMBDA OUTSIDE RANGE O
     1F SUBROUTINE GLNGTH.   SLMDA = ',E12.5,'***********')
      INDIC=-1
      GO TO 99
   70 IF(SLMDA.GE.2.07044)GO TO 71
      A=1.99442
      B=0.274849
      GO TO 95
   71 IF(SLMDA.GE.2.15453)GO TO 72
      A=1.99287
      B=0.280545
      GO TO 95
   72 IF(SLMDA.GE.2.24767)GO TO 73
      A=1.989503
      B=0.286635
      GO TO 95
   73 IF(SLMDA.GE.2.35038)GO TO 74
      A=1.98507
      B=0.291881
      GO TO 95
   74 IF(SLMDA.GE.2.46373)GO TO 75
      A=1.97853
      B=0.297581
      GO TO 95
   75 IF(SLMDA.GE.2.58781)GO TO 76
      A=1.97175
      B=0.302239
      GO TO 95
   76 IF(SLMDA.GE.2.72339)GO TO 77
      A=1.96328
      B=0.307023
      GO TO 95
   77 IF(SLMDA.GE.2.87037)GO TO 78
      A=1.95482
      B=0.311076
      GO TO 95
   78 IF(SLMDA.GE.3.02894)GO TO 79
      A=1.94521
      B=0.315067
      GO TO 95
```

```
79 IF(SLMDA.GE.3.19819)GO TO 80
   A=1.93701
   B=0.318071
   GO TO 95
80 IF(SLMDA.GE.3.37780)GO TO 81
   A=1.92864
   B=0.320819
   GO TO 95
81 IF(SLMDA.GE.3.56703)GO TO 82
   A=1.92034
   B=0.323288
   GO TO 95
82 IF(SLMDA.GE.3.76484)GO TO 83
   A=1.91325
   B=0.325220
   GO TO 95
83 IF(SLMDA.GT.4.0)GO TO 91
   A=1.90667
   B=0.326875
95 GP=(SLMDA/A)**(1./B)
99 RETURN
   END
```

APPENDIX B

Balloon Design Program - Input

The input data for the program consists of three cards per balloon to be designed. In order to stop the program without generating an error statement a similar set of three cards with no data is required.

Card 1. Format (F10.0, I10, F10.0, I10, F10.4, I10, F10.0, I10)

Columns	Variable	Description
1-10	P	Design payload in pounds
20	KEY	Altitude option: 1-Altitude in feet 2-Altitude in millibars
21-30	CONST	Design altitude in feet or millibars
40	CODEF	Film type: 1-Polyethelene; 2-Mylar
41-50	FTHICK	Film thickness in inches
60	CODET	Load tape type: 1-Polyester; 2-Kevlar
61-70	TLR	Tape load rating in pounds
71-80	NT	Number of load tapes

Card 2. Format (F10.2, F10.2, F10.2, F10.2, I10, F10.2, F10.2, F10.4)

Columns	Variable	Description
1-10	TL	Top load in pounds, (+) up, (-) down
11-20	TAUO	Stress constant, 0. for natural shape
21-30	TAUI	Stress constant, 0. for natural shape
31-40	ALPHA	Superpressure in feet
41-50	N	Print increment, 0 for all points
51-60	DSO	Nondimensional gore increment (ds/λ)
61-70	CSTART	Nondimensional gore position of edge of cap (S_{cap}/λ)
71-80	CAP	Film thickness including cap in inches

Card 3. Format (3I10, 2F10.0)

Columns	Variable	Description
10	KEY2	Output control (0-print only, 1-punch deck & print, 2-disk file generated)
11-20	MPT	Number of points desired in load-altitude curve
30	IGAS	Identify lifting gas, 1-Helium
31-40	PMIN	Minimum Recommended payload in pounds
41-50	PMAX	Maximum Recommended payload in pounds

The output of this program consists of a). the nondimensional shape parameters normalized with respect to λ; b). the dimensional manufacturers table coordinates; c) 100 equally spaced points needed for input into the analysis program; and d). the load-altitude curve. A sample of the printed output is as follows:

FILM TYPE	: POLYETHYLENE
FILM THICKNESS	: 0.0005 IN
TAPE TYPE	: POLYESTER
TAPE LOAD RATING	: 500. LBS
NUMBER OF TAPES	: 91
PAYLOAD	: 4030. LBS
(INCL TOPLOAD OF)	: -30. LBS
DESIGN ALTITUDE	: 100000. FT

NON-DIMENSIONAL QUANTITIES

SIGMA FILM	= 0.0301
EM	= 38.771
EC	= 38.771
EMC	= 19.386
SIGMA TAPE	= 0.0294
KT	= 162.6

MISC. PARAMETERS

TAU0	= 0.0
TAU1	= 0.0
ALPHA	= 0.0
DS0	= 0.025
CSTART	= 1.20
N	= 0

ITERATION RECORD (INITIAL AND FINAL ANGLES)

52.47168	-88.59433
54.29198	-90.23912
54.28452	-90.22377

MANUFACTURER'S TABLE LAYOUT

S	R	Z	T	TAUM	TAUC	GORE POSITION (FT)	HALF GORE WIDTH (IN)
0.0	0.0	0.0	1.70030	0.0	0.0	0.0	0.0
0.02500	0.02030	0.01460	1.70074	0.297	0.0	4.1	1.39
0.05000	0.04059	0.02920	1.70121	0.291	0.0	8.2	2.79
0.07500	0.06087	0.04382	1.70172	0.265	0.0	12.2	4.18
0.10000	0.08114	0.05845	1.70226	0.279	0.0	16.3	5.57
0.12500	0.10140	0.07310	1.70282	0.273	0.0	20.4	6.96
0.15000	0.12164	0.08777	1.70342	0.268	0.0	24.5	8.35
0.17500	0.14186	0.10247	1.70405	0.263	0.0	28.6	9.73
0.20000	0.16205	0.11721	1.70472	0.258	0.0	32.7	11.12
0.22500	0.18221	0.13200	1.70541	0.253	0.0	36.7	12.50
0.25000	0.20233	0.14684	1.70614	0.249	0.0	40.8	13.88
0.27500	0.22240	0.16175	1.70690	0.244	0.0	44.9	15.25
0.30000	0.24241	0.17673	1.70770	0.240	0.0	49.0	16.62
0.32500	0.26235	0.19181	1.70854	0.236	0.0	53.1	17.99
0.35000	0.28221	0.20700	1.70940	0.232	0.0	57.2	19.35
0.37500	0.30197	0.22230	1.71031	0.228	0.0	61.3	20.70
0.40000	0.32164	0.23774	1.71126	0.225	0.0	65.4	22.05
0.42500	0.34118	0.25333	1.71225	0.221	0.0	69.4	23.39
0.45000	0.36059	0.26909	1.71328	0.218	0.0	73.5	24.72
0.47500	0.37984	0.28504	1.71435	0.215	0.0	77.6	26.04
0.50000	0.39892	0.30119	1.71547	0.212	0.0	81.7	27.34
0.52500	0.41781	0.31756	1.71664	0.209	0.0	85.8	28.64
0.55000	0.43649	0.33418	1.71785	0.206	0.0	89.9	29.91
0.57500	0.45493	0.35106	1.71912	0.203	0.0	94.0	31.18
0.60000	0.47311	0.36822	1.72044	0.201	0.0	98.1	32.42
0.62500	0.49099	0.38569	1.72182	0.198	0.0	102.2	33.65
0.65000	0.50856	0.40347	1.72325	0.196	0.0	106.3	34.85
0.67500	0.52578	0.42160	1.72474	0.194	0.0	110.4	36.03
0.70000	0.54261	0.44008	1.72630	0.192	0.0	114.5	37.18
0.72500	0.55903	0.45894	1.72792	0.190	0.0	118.6	38.30
0.75000	0.57498	0.47818	1.72960	0.188	0.0	122.7	39.39
0.77500	0.59043	0.49784	1.73136	0.186	0.0	126.8	40.45
0.80000	0.60533	0.51791	1.73317	0.184	0.0	130.8	41.47
0.82500	0.61965	0.53840	1.73506	0.183	0.0	134.9	42.45
0.85000	0.63332	0.55933	1.73702	0.181	0.0	139.0	43.39

0.87500	0.64630	0.58070	1.73905	0.180	0.0	143.1	44.27
0.90000	0.65853	0.60250	1.74115	0.179	0.0	147.2	45.11
0.92500	0.66996	0.62473	1.74332	0.178	0.0	151.3	45.89
0.95000	0.68053	0.64738	1.74555	0.177	0.0	155.4	46.62
0.97500	0.69018	0.67044	1.74785	0.176	0.0	159.5	47.26
1.00000	0.69885	0.69389	1.75021	0.175	0.0	163.6	47.87
1.02500	0.70648	0.71769	1.75262	0.175	0.0	167.7	48.39
1.05000	0.71301	0.74182	1.75509	0.174	0.0	171.8	48.84
1.07500	0.71838	0.76624	1.75760	0.174	0.0	175.9	49.21
1.10000	0.72254	0.79089	1.76014	0.174	0.0	180.0	49.49
1.12500	0.72544	0.81571	1.76272	0.174	0.0	184.1	49.69
1.15000	0.72701	0.84066	1.76531	0.174	0.0	188.2	49.80
1.17500	0.72723	0.86566	1.76790	0.174	0.0	192.3	49.81
1.20000	0.72605	0.89063	1.77050	0.174	0.0	196.4	49.73

CAP STARTS AT S = 1.20 ... SIGMA CAP = 0.1385

1.22500	0.72345	0.91549	1.77308	0.175	0.0	200.5	49.55
1.25000	0.71940	0.94015	1.78224	0.176	0.0	204.6	49.28
1.27499	0.71392	0.96454	1.79125	0.178	0.0	208.7	48.90
1.29999	0.70700	0.98856	1.80005	0.179	0.0	212.8	48.43
1.32499	0.69867	1.01213	1.80860	0.181	0.0	216.9	47.86
1.34999	0.68895	1.03515	1.81686	0.183	0.0	221.0	47.20
1.37499	0.67788	1.05756	1.82479	0.185	0.0	225.1	46.44
1.39999	0.66550	1.07927	1.83235	0.187	0.0	229.2	45.59
1.42499	0.65185	1.10021	1.83951	0.190	0.0	233.2	44.66
1.44999	0.63700	1.12032	1.84625	0.192	0.0	237.3	43.65
1.47499	0.62101	1.13953	1.85256	0.195	0.0	241.4	42.55
1.49999	0.60395	1.15780	1.85841	0.198	0.0	245.5	41.39
1.52499	0.58590	1.17509	1.86380	0.201	0.0	249.6	40.15
1.54999	0.56692	1.19135	1.86874	0.204	0.0	253.7	38.85
1.57499	0.54710	1.20658	1.87321	0.207	0.0	257.8	37.50
1.59999	0.52650	1.22075	1.87725	0.211	0.0	261.9	36.09
1.62499	0.50522	1.23386	1.88085	0.215	0.0	266.0	34.63
1.64999	0.48332	1.24591	1.88404	0.218	0.0	270.1	33.13
1.67499	0.46088	1.25692	1.88683	0.222	0.0	274.2	31.59
1.69999	0.43796	1.26690	1.88926	0.226	0.0	278.3	30.03
1.72499	0.41463	1.27588	1.89135	0.231	0.0	282.3	28.43
1.74999	0.39095	1.28390	1.89313	0.235	0.0	286.4	26.81
1.77499	0.36698	1.29099	1.89462	0.240	0.0	290.5	25.17
1.79999	0.34277	1.29719	1.89585	0.245	0.0	294.6	23.51
1.82499	0.31835	1.30256	1.89686	0.250	0.0	298.7	21.83
1.84999	0.29378	1.30714	1.89766	0.255	0.0	302.8	20.15
1.87499	0.26908	1.31100	1.89830	0.261	0.0	306.9	18.46
1.89999	0.24428	1.31419	1.89878	0.266	0.0	310.9	16.76
1.92498	0.21942	1.31676	1.89915	0.272	0.0	315.0	15.05
1.94998	0.19450	1.31878	1.89941	0.279	0.0	319.1	13.35
1.97498	0.16955	1.32032	1.89959	0.285	0.0	323.2	11.64
1.99998	0.14457	1.32143	1.89970	0.292	0.0	327.3	9.92
2.02498	0.11958	1.32218	1.89977	0.299	0.0	331.3	8.21
2.04998	0.09459	1.32263	1.89981	0.307	0.0	335.4	6.49
2.07498	0.06959	1.32284	1.89983	0.315	0.0	339.5	4.78
2.09998	0.04459	1.32289	1.89983	0.323	0.0	343.6	3.06
2.12498	0.01959	1.32283	1.89982	0.332	0.0	347.7	1.35
2.14457	0.00000	1.32275	1.89982	0.0	0.0	350.8	0.00

FINAL QUANTITIES (LAMBDA = 164.8 FT):

VOLUME	= 0.584E 07 FT3	INITIAL ANGLE	= 54.285 DEG
AREA	= 0.160E 06 FT2	FINAL ANGLE	=-90.225 DEG

FILM WEIGHT = 965. LBS
TAPE WEIGHT = 262. LBS
TOT. WEIGHT = 1227. LBS

TOP LOAD REACTION = 30.058 LBS

GP(1) =	0.0	GW(1) =	0.0	GWT(1) =	0.005	GT(1) =	0.000
GP(2) =	3.544	GW(2) =	1.210	GWT(2) =	0.004	GT(2) =	0.000
GP(3) =	7.088	GW(3) =	2.420	GWT(3) =	0.005	GT(3) =	0.000
GP(4) =	10.632	GW(4) =	3.628	GWT(4) =	0.006	GT(4) =	0.000
GP(5) =	14.176	GW(5) =	4.836	GWT(5) =	0.006	GT(5) =	0.000
GP(6) =	17.720	GW(6) =	6.043	GWT(6) =	0.006	GT(6) =	0.000
GP(7) =	21.263	GW(7) =	7.249	GWT(7) =	0.006	GT(7) =	0.000
GP(8) =	24.607	GW(8) =	8.453	GWT(8) =	0.007	GT(8) =	0.000
GP(9) =	28.351	GW(9) =	9.656	GWT(9) =	0.007	GT(9) =	0.000
GP(10) =	31.895	GW(10) =	10.858	GWT(10) =	0.007	GT(10) =	0.000
GP(11) =	35.439	GW(11) =	12.057	GWT(11) =	0.008	GT(11) =	0.000
GP(12) =	38.983	GW(12) =	13.253	GWT(12) =	0.006	GT(12) =	0.000
GP(13) =	42.527	GW(13) =	14.448	GWT(13) =	0.008	GT(13) =	0.000
GP(14) =	46.071	GW(14) =	15.639	GWT(14) =	0.008	GT(14) =	0.000
GP(15) =	49.615	GW(15) =	16.827	GWT(15) =	0.009	GT(15) =	0.000
GP(16) =	53.159	GW(16) =	18.011	GWT(16) =	0.009	GT(16) =	0.000
GP(17) =	56.702	GW(17) =	19.190	GWT(17) =	0.009	GT(17) =	0.000
GP(18) =	60.246	GW(18) =	20.365	GWT(18) =	0.010	GT(18) =	0.000
GP(19) =	63.790	GW(19) =	21.534	GWT(19) =	0.010	GT(19) =	0.000
GP(20) =	67.334	GW(20) =	22.697	GWT(20) =	0.010	GT(20) =	0.000
GP(21) =	70.878	GW(21) =	23.853	GWT(21) =	0.010	GT(21) =	0.000
GP(22) =	74.422	GW(22) =	25.002	GWT(22) =	0.011	GT(22) =	0.000
GP(23) =	77.966	GW(23) =	26.143	GWT(23) =	0.011	GT(23) =	0.000
GP(24) =	81.510	GW(24) =	27.275	GWT(24) =	0.011	GT(24) =	0.000
GP(25) =	85.054	GW(25) =	28.396	GWT(25) =	0.011	GT(25) =	0.000
GP(26) =	68.598	GW(26) =	29.506	GWT(26) =	0.012	GT(26) =	0.000
GP(27) =	92.141	GW(27) =	30.604	GWT(27) =	0.012	GT(27) =	0.000
GP(28) =	95.685	GW(28) =	31.690	GWT(28) =	0.012	GT(28) =	0.000
GP(29) =	99.229	GW(29) =	32.762	GWT(29) =	0.012	GT(29) =	0.000
GP(30) =	102.773	GW(30) =	33.818	GWT(30) =	0.013	GT(30) =	0.000
GP(31) =	106.317	GW(31) =	34.859	GWT(31) =	0.013	GT(31) =	0.000
GP(32) =	109.861	GW(32) =	35.879	GWT(32) =	0.013	GT(32) =	0.000
GP(33) =	113.405	GW(33) =	36.880	GWT(33) =	0.013	GT(33) =	0.000
GP(34) =	116.949	GW(34) =	37.860	GWT(34) =	0.014	GT(34) =	0.000
GP(35) =	120.493	GW(35) =	38.617	GWT(35) =	0.014	GT(35) =	0.000
GP(36) =	124.036	GW(36) =	39.750	GWT(36) =	0.014	GT(36) =	0.000
GP(37) =	127.580	GW(37) =	40.658	GWT(37) =	0.014	GT(37) =	0.000
GP(38) =	131.124	GW(38) =	41.538	GWT(38) =	0.015	GT(38) =	0.000
GP(39) =	134.668	GW(39) =	42.386	GWT(39) =	0.015	GT(39) =	0.000
GP(40) =	138.212	GW(40) =	43.198	GWT(40) =	0.015	GT(40) =	0.000
GP(41) =	141.756	GW(41) =	43.976	GWT(41) =	0.015	GT(41) =	0.000
GP(42) =	145.300	GW(42) =	44.717	GWT(42) =	0.015	GT(42) =	0.000
GP(43) =	148.844	GW(43) =	45.420	GWT(43) =	0.016	GT(43) =	0.000
GP(44) =	152.388	GW(44) =	46.081	GWT(44) =	0.016	GT(44) =	0.000
GP(45) =	155.932	GW(45) =	46.699	GWT(45) =	0.016	GT(45) =	0.000
GP(46) =	159.475	GW(46) =	47.270	GWT(46) =	0.016	GT(46) =	0.000
GP(47) =	163.019	GW(47) =	47.784	GWT(47) =	0.016	GT(47) =	0.000
GP(48) =	166.563	GW(48) =	48.246	GWT(48) =	0.016	GT(48) =	0.000
GP(49) =	170.107	GW(49) =	48.654	GWT(49) =	0.016	GT(49) =	0.000
GP(50) =	173.651	GW(50) =	49.005	GWT(50) =	0.016	GT(50) =	0.000
GP(51) =	177.195	GW(51) =	49.297	GWT(51) =	0.016	GT(51) =	0.000
GP(52) =	180.739	GW(52) =	49.527	GWT(52) =	0.016	GT(52) =	0.000
GP(53) =	184.283	GW(53) =	49.695	GWT(53) =	0.017	GT(53) =	0.000
GP(54) =	187.827	GW(54) =	49.788	GWT(54) =	0.017	GT(54) =	0.000
GP(55) =	191.371	GW(55) =	49.810	GWT(55) =	0.017	GT(55) =	0.000
GP(56) =	194.914	GW(56) =	49.761	GWT(56) =	0.017	GT(56) =	0.000
GP(57) =	198.458	GW(57) =	49.642	GWT(57) =	0.017	GT(57) =	0.000

GP(58) =	202.002	GW(58) =	49.452	GWT(58) =	0.032	GT(58) =	0.000
GP(59) =	205.546	GW(59) =	49.190	GWT(59) =	0.059	GT(59) =	0.000
GP(60) =	209.090	GW(60) =	48.856	GWT(60) =	0.059	GT(60) =	0.000
GP(61) =	212.634	GW(61) =	48.447	GWT(61) =	0.058	GT(61) =	0.000
GP(62) =	216.178	GW(62) =	47.958	GWT(62) =	0.058	GT(62) =	0.000
GP(63) =	219.722	GW(63) =	47.400	GWT(63) =	0.057	GT(63) =	0.000
GP(64) =	223.266	GW(64) =	46.773	GWT(64) =	0.056	GT(64) =	0.000
GP(65) =	226.810	GW(65) =	46.079	GWT(65) =	0.055	GT(65) =	0.000
GP(66) =	230.353	GW(66) =	45.321	GWT(66) =	0.055	GT(66) =	0.000
GP(67) =	233.897	GW(67) =	44.500	GWT(67) =	0.054	GT(67) =	0.000
GP(68) =	237.441	GW(68) =	43.619	GWT(68) =	0.053	GT(68) =	0.000
GP(69) =	240.985	GW(69) =	42.673	GWT(69) =	0.052	GT(69) =	0.000
GP(70) =	244.529	GW(70) =	41.670	GWT(70) =	0.051	GT(70) =	0.000
GP(71) =	248.073	GW(71) =	40.617	GWT(71) =	0.049	GT(71) =	0.000
GP(72) =	251.617	GW(72) =	39.517	GWT(72) =	0.048	GT(72) =	0.000
GP(73) =	255.161	GW(73) =	38.372	GWT(73) =	0.047	GT(73) =	0.000
GP(74) =	258.705	GW(74) =	37.185	GWT(74) =	0.046	GT(74) =	0.000
GP(75) =	262.248	GW(75) =	35.961	GWT(75) =	0.044	GT(75) =	0.000
GP(76) =	265.792	GW(76) =	34.699	GWT(76) =	0.043	GT(76) =	0.000
GP(77) =	269.336	GW(77) =	33.402	GWT(77) =	0.042	GT(77) =	0.000
GP(78) =	272.879	GW(78) =	32.078	GWT(78) =	0.040	GT(78) =	0.000
GP(79) =	276.423	GW(79) =	30.729	GWT(79) =	0.039	GT(79) =	0.000
GP(80) =	279.967	GW(80) =	29.357	GWT(80) =	0.037	GT(80) =	0.000
GP(81) =	283.510	GW(81) =	27.966	GWT(81) =	0.036	GT(81) =	0.000
GP(82) =	287.054	GW(82) =	26.557	GWT(82) =	0.034	GT(82) =	0.000
GP(83) =	290.598	GW(83) =	25.133	GWT(83) =	0.033	GT(83) =	0.000
GP(84) =	294.142	GW(84) =	23.695	GWT(84) =	0.031	GT(84) =	0.000
GP(85) =	297.685	GW(85) =	22.246	GWT(85) =	0.029	GT(85) =	0.000
GP(86) =	301.229	GW(86) =	20.789	GWT(86) =	0.028	GT(86) =	0.000
GP(87) =	304.773	GW(87) =	19.324	GWT(87) =	0.026	GT(87) =	0.000
GP(88) =	308.316	GW(88) =	17.853	GWT(88) =	0.025	GT(88) =	0.000
GP(89) =	311.860	GW(89) =	16.377	GWT(89) =	0.023	GT(89) =	0.000
GP(90) =	315.404	GW(90) =	14.897	GWT(90) =	0.021	GT(90) =	0.000
GP(91) =	318.948	GW(91) =	13.414	GWT(91) =	0.020	GT(91) =	0.000
GP(92) =	322.491	GW(92) =	11.929	GWT(92) =	0.018	GT(92) =	0.000
GP(93) =	326.035	GW(93) =	10.442	GWT(93) =	0.016	GT(93) =	0.000
GP(94) =	329.579	GW(94) =	8.953	GWT(94) =	0.015	GT(94) =	0.000
GP(95) =	333.122	GW(95) =	7.463	GWT(95) =	0.013	GT(95) =	0.000
GP(96) =	336.666	GW(96) =	5.973	GWT(96) =	0.012	GT(96) =	0.000
GP(97) =	340.210	GW(97) =	4.481	GWT(97) =	0.010	GT(97) =	0.000
GP(98) =	343.753	GW(98) =	2.989	GWT(98) =	0.008	GT(98) =	0.000
GP(99) =	347.297	GW(99) =	1.496	GWT(99) =	0.007	GT(99) =	0.000
GP(100) =	350.841	GW(100) =	0.002	GWT(100) =	0.005	GT(100) =	0.000

LOAD - ALTITUDE DATA

GROSS AIRBORN WEIGHT - (KG)	ALTITUDE - (KM)
0.2386813E 04	0.3045546E 02
0.2398161E 04	0.3042964E 02
0.2409509E 04	0.3040402E 02
0.2420859E 04	0.3037819E 02
0.2432207E 04	0.3035216E 02
0.2443556E 04	0.3032632E 02
0.2454904E 04	0.3030070E 02
0.2466252E 04	0.3027509E 02
0.2477602E 04	0.3024968E 02
0.2488950E 04	0.3022426E 02
0.2500298E 04	0.3019905E 02
0.2511647E 04	0.3017384E 02
0.2522995E 04	0.3014865E 02
0.2534345E 04	0.3012405E 02
0.2545693E 04	0.3009926E 02
0.2557041E 04	0.3007446E 02
0.2568390E 04	0.3005008E 02
0.2579738E 04	0.3002550E 02
0.2591088E 04	0.3000133E 02
0.2602436E 04	0.2997694E 02
0.2613784E 04	0.2995297E 02
0.2625133E 04	0.2992880E 02
0.2636481E 04	0.2990504E 02
0.2647831E 04	0.2988129E 02
0.2659179E 04	0.2985753E 02
0.2670527E 04	0.2983397E 02
0.2681875E 04	0.2981041E 02
0.2693224E 04	0.2978706E 02
0.2704574E 04	0.2976372E 02
0.2715922E 04	0.2974059E 02
0.2727270E 04	0.2971744E 02
0.2738618E 04	0.2969450E 02
0.2749967E 04	0.2967157E 02
0.2761316E 04	0.2964885E 02
0.2772665E 04	0.2962611E 02
0.2784013E 04	0.2960359E 02
0.2795361E 04	0.2958107E 02
0.2806709E 04	0.2955835E 02
0.2818059E 04	0.2953561E 02
0.2829408E 04	0.2951309E 02
0.2840756E 04	0.2949059E 02

APPENDIX C

Balloon Analysis Program

BALAN

```
//$OPTIONS
C*************************************************************************
C                            - B A L A N -
C                       BALLOON ANALYSIS PROGRAM
C*************************************************************************
C        THIS PROGRAM IS INTENDED TO COMPUTE THE SHAPE AND STRESS
C        DISTRIBUTIONS OF A BALLOON AT ANY ALTITUDE WHERE THE FILM
C        MAY BE CONSIDERED TO LOBE BETWEEN TAPES.  THE MANUFACTURED
C        RADIUS AND WEIGHT DISTRIBUTIONS ARE REQUIRED INPUTS.  THE
C        DESIGN SHAPE PARAMETERS ARE NEEDED TO START THE ITERATION
C        PROCESS.  THE OUTPUT CONSISTS OF THE DEFORMED SHAPE PARA-
C        METERS, MERIDIONAL STRESS, CIRCUMFERENTIAL STRESS, SHEAR
C        STRESS, PRINCIPAL STRESSES, AND TENSION FIELD PATTERNS
C        WHERE APPLICABLE.
C        ALL QUANTITIES ARE NONDIMENSIONALIZED WITH RESPECT TO
C        SMAX, THE UNDEFORMED GORE LENGTH, AND PD, THE DEISGN
C        PAYLOAD.
C*************************************************************************
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(300),C(1801),F(1801),X(1801),RO(300),WT
     1(300),GP(300),WTAPE(300),SMP(300),SCP(300)
      REAL*8  NU,KBAR,LAMBDA
      ATAN(Z)=DATAN(Z)
      ARSIN(Z)=DARSIN(Z)
      ARCOS(Z)=DARCOS(Z)
      SIN(Z)=DSIN(Z)
      COS(Z)=DCOS(Z)
      TAN(Z)=DTAN(Z)
      READ(5,800) PD,PTOP,WBAL,SMAX,LAMBDA
      READ(5,801) NU,KBAR,EBAR,N
      READ(5,805) KMAX,ABAR,BBAR
      READ(5,802) (GP(K),RO(K),WT(K),WTAPE(K),K=1,KMAX)
      SMAX=GP(KMAX)
      SOL=SMAX/LAMBDA
      IMAX=6*KMAX+1
      DO 10 K=1,KMAX
      L=6*(K-1)+1
      READ(5,803) (X(L+I), I=2,6)
      X(L+1)=0.0
   10 CONTINUE
      PI=4.*ATAN(1.0D0)
      PON=PI/N
      X(1)=1.-PTOP
      BBAR=BBAR*SOL**3
      KBAR=KBAR/2./PI
      EBAR=EBAR*SOL
      VTOP=X(IMAX)
      DO 20 K=1,KMAX
      L=6*(K-1)+1
      X(L+2)=X(L+2)/SOL
      X(L+3)=X(L+3)/SOL
      X(L+6)=(VTOP-X(L+6))/SOL**3*PI
      GP(K)=GP(K)/SMAX
      RO(K)=RO(K)*N/12./PI/SMAX
      WT(K)=WT(K)*SOL
      WTAPE(K)=WTAPE(K)*SOL
   20 CONTINUE
      WRITE(6,903)(K,GP(K),RO(K),WT(K),WTAPE(K),K=1,KMAX)
      SPON=SIN(PON)
      CPON=COS(PON)
```

```
      S2PON=SPON*SPON
      DS=1./(KMAX-1)
      ZPA=ABAR
      DO 3 MNN=1,5
100   DO 200 K=1,KMAX
      IF(K.EQ.50)EBAR=4.6*EBAR
      EM=EBAR/(1.-NU**2)
      EC=EM
      EMC=EC*NU
      D=EM*EC-EMC**2
      DM=EC/D
      DC=EM/D
      DMC=-EMC/D
      L=6*(K-1)+1
      STH=SIN(X(L+5))
      CTH=COS(X(L+5))
      SB=SIN(X(L+1))
      CB=COS(X(L+1))
      ALFA=ARSIN(SPON*CTH)
      IF(ZPA.LT.0.)ALFA=-ALFA
      IF(CTH.LT.0.)ALFA=-ALFA
      ABET=ALFA+X(L+1)
      PHI=2.*ABET
      SBPON=SIN(ABET)
      FIR=RO(K)*DC/DMC+(DC*DM/DMC-DMC)*KBAR
      IF(K.EQ.1)THP=(X(12)-X(6))/DS
      IF(K.EQ.KMAX)THP=(X(L+5)-X(L-1))/DS
      IF(K.EQ.1) GO TO 102
      IF(K.EQ.KMAX) GO TO 102
      THP=(X(L+11)-X(L-1))/2./DS
102   CONTINUE
      ZPA=X(L+3)+ABAR
      SM=(X(L+4)-KBAR*DMC*ABET/PON*(RO(K)*BBAR*ZPA+STH*(WT(K)
     1-WTAPE(K)/PON/2.)))/(RO(K)+KBAR*DM+KBAR*DMC*ABET/PON*
     2RO(K)*THP)
      EF=X(L+4)-RO(K)*SM
      IF(EF.LT.0.)SM=X(L+4)/RO(K)
      IF(EF.LT.0.)EF=0.
      FTB=SPON*CB*CTH+SB*(CPON*CTH*CTH-1.+STH*STH)
      SC=ABET/PON*(RO(K)*SM*THP                        +RO(K)*BBAR
     1*ZPA+STH*(WT(K)-WTAPE(K)/PON/2.))
      SMP(K)=SM
      A(K)=(X(L+4)-RO(K)*SM)*2.*PON
105   CONTINUE
      IF(SC.LT.0.)SC=0.
      SCP(K)=SC
      EMP1=1.+DM*SM+DMC*SC
      ECP1=1.+DMC*SM+DC*SC
      F1B=SPON*CB+SB*CTH*(CPON-1.)
      F2B=FTB/PON
      IF(EF.EQ.0.)BET=ARSIN(-PON*STH*WTAPE(K)/SC)
      IF(EF.EQ.0.)STUFF=TAN(BET)
      IF(EF.EQ.0.)GO TO 104
      RATIO=RO(K)*SM/(X(L+4)-RO(K)*SM)
      TOP=BBAR*ZPA*X(L+2)*EMP1*SPON+PON*(WT(K)-WTAPE(K)
     1/PON/2.)*STH-SC*SPON*CTH*CB-RATIO*WTAPE(K)*STH/2.
      BOT=CB*(RATIO*SC+SC*(CPON*CTH*CTH+STH*STH))
      IF(SC.EQ.0.)STUFF=99999999
      IF(SC.EQ.0.)GO TO 104
      STUFF=TOP/BOT
104   CONTINUE
```

```
          ASEG=(PHI-SIN(PHI))*(SPUN/SBPUN)**2*CTH
          IF(ZPA.EQ.0.)SIGN=1.
          IF(ZPA.EQ.0.) GO TO 101
          SIGN=ZPA/DABS(ZPA)
      101 ASUBN=N*(SIN(2*PUN)+SIGN*ASEG)
          NN=1
          JJ=0
          IF(K.EQ.1) GO TO 170
          IF(K.EQ.KMAX)GO TO 180
          IF(K.EQ.2) GO TO 160
      159 IF(K.GE.2)JJ=6
          IF(K.GE.2)NN=2
      160 CONTINUE
          C(L+1)=ATAN(STUFF)
          IF(K.EQ.KMAX-1)GO TO 161
          C(L+2+JJ)=X(L-4)+NN*DS*STH*EMPI
          C(L+3+JJ)=X(L-3)+NN*DS*CTH*EMPI
          C(L+4+JJ)=X(L-2)+NN*DS*(WT(K)*CTH+SC*STH/PUN*FIB)
          IF(JJ.EQ.6)GO TO 161
          IF(K.EQ.2) GO TO 159
      161 C(L+5-JJ)=X(L+11)+NN*DS/X(L+4)*(WT(K)*STH+BBAR*ZPA*X(L+2)*EMPI
         1*SPUN/PUN-SC/PUN*FTB)
          C(L+6-JJ)=X(L+12)+NN*DS*EMPI*X(L+2)*X(L+2)*ASUBN*CTH/2.
          IF(JJ.EQ.0) GO TO 200
          IF(K.EQ.KMAX-1)NN=1
          IF(K.EQ.KMAX-1)JJ=0
          IF(K.EQ.KMAX-1) GO TO 161
          GO TO 200
      170 CONTINUE
          C(1)=BBAR*X(7)-PTOP-WBAL
          C(2)=ATAN(STUFF)
          C(3)=RO(1)
          C(4)=0.
          C(5)=X(1)/CTH/2./PI
          C(6)=X(12)+DS/X(5)*(STH*WT(K)+BBAR*ZPA*X(3)*EMPI*SPON/PUN
         1-SC*F2B)
          C(7)=X(13)+DS*EMPI*X(3)*X(3)/2.*CTH*ASUBN
          GO TO 200
      180 CONTINUE
          C(L+1)=ATAN(STUFF)
          C(L+2)=RO(KMAX)
          C(L+3)=X(L-3)+DS*CTH*EMPI
          C(L+4)=X(L-2)+DS*(WT(K)*CTH+SC*STH/PUN*FIB)
          C(L+5)=-PI/2.
          PR=BBAR*ZPA*RO(KMAX)**2*PI
          PSOP=PTOP-PR
          IF(PSOP.GT.0.)C(L+5)=-ARCOS((PR-PTOP)/2./PI/X(L+4))
          IF(PSOP.LT.0.)C(L+5)=ARCOS((-PR+PTOP)/2./PI/X(L+4))
          C(L+6)=0.
      200 CONTINUE
          EBAR=EBAR/4.6
          RMS=0.
          SIGMA=.50
          DO 210 I=1,IMAX
          F(I)=C(I)-X(I)
          X(I)=X(I)+SIGMA*F(I)
          RMS=RMS+F(I)*F(I)
      210 CONTINUE
        2 CONTINUE
          DO 1 KL=1,KMAX
          IF(KL.EQ.1) WRITE(6,13) KL,X(KL),F(KL)
```

```
      WRITE(6,7) KL,(X(I+6*KL-5),I=1,5),(F(I+6*KL-5),I=1,5),
     1SMP(KL),SCP(KL)
   13 FORMAT(I5,2F10.6)
    7 FORMAT(I5,12F10.6)
    1 CONTINUE
      CHECK=DABS(F(1)/X(1))
      RMS=RMS**.5
      WRITE(6,900)RMS
      HT=BBAR/SOL**3
      VOL=X(7)*SMAX**3
      PAY=X(1)*PD
    3 CONTINUE
      WRITE(6,908) PAY,LAMBDA,EBAR,VOL,NU,ABAR,SMAX,
     1KBAR,HT,N,KMAX
      CALL PRINT (KMAX,A,SMP,SCP,X,WTAPE,DS)
C     FORMAT STATEMENTS FOR INPUT
C
  800 FORMAT(5E15.7)
  801 FORMAT(3E15.7,I15)
  802 FORMAT(4E15.7)
  803 FORMAT(5E15.7)
  805 FORMAT(I15,2E15.7)
C
C     FORMAT STATEMENTS FOR OUTPUT
C
  900 FORMAT(1X,'RMS VALUE = ',E20.13)
  903 FORMAT(I4,4E15.7)
  908 FORMAT(1H1,10X,'DESIGN VALUES',/,10X,'PAYLOAD=',6X,F7.2,
     1' LBS',10X,'LAMBDA=',F4.0,5X,'EBAR=',F7.2,/,10X,'VOLUME=',
     2E14.4,' FT(3)',8X,'NU=',F8.2,5X,'ABAR=',F5.2,/,10X,'GORE LENGTH=',
     3F9.2,' FT',11X,'KBAR=',F6.2,5X,'BBAR=',F5.2,/,10X,
     4'NUMBER OF TAPES=',I5,/,10X,'NUMBER OF POINTS=',I4,//)
  999 STOP
      END

      SUBROUTINE PRINT (KMAX,F,SMP,SCP,X,WTAPE,DS)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION F(300),SCP(300),SMP(300),X(1801),WTAPE(300)
      IMAX=6*KMAX+1
      WRITE(6,801)
      WRITE(6,800) (X(I),I=2,IMAX)
      WRITE(6,900)
      PI=4.*DATAN(1.D0)
      DO 105 K=1,KMAX
C        CALCULATE SHEAR STRESS
C
      CTH=DCOS(X(6*K))
      IF(K.EQ.1) GO TO 101
      IF(K.EQ.KMAX) GO TO 102
      TAU=(F(K+1)-F(K-1))/4./DS-WTAPE(K)*CTH/2.
      GO TO 103
  101 TAU=(-3.*F(K)+4.*F(K+1)-F(K+2))/4./DS-WTAPE(K)*CTH/2.
      GO TO 103
  102 TAU=(F(K-2)-4.*F(K-1)+3.*F(K))/4./DS-WTAPE(K)*CTH/2.
  103 SP=SMP(K)+SCP(K)
C        CHECK TO SEE IF A TENSION FIELD EXISTS
C
      IF(SCP(K).LE.0.) GO TO 104
      TAUMAX=DSQRT(((SMP(K)-SCP(K))/2.D0)**2+TAU**2)
      S1=SP/2.+TAUMAX
      S2=SP/2.-TAUMAX
```

```
          ANG=DATAN(2.D0*TAU/(SMP(K)-SCP(K)))/2.
          IF(SCP(K).GT.SMP(K)) ANG=ANG+PI/2.D0
          GO TO 105
    104   ANG=-DATAN(SMP(K)/TAU)-PI/2.D0
          S1=SMP(K)/DCOS(ANG)/DCOS(ANG)
          S2=0.
          TAUMAX=SMP(K)/2.D0
    105   WRITE(6,901) K,F(K),SMP(K),SCP(K),TAU,S1,S2,TAUMAX,ANG
C
C           FORMAT STATEMENTS FOR OUTPUT
C
    800   FORMAT(6F10.4)
    801   FORMAT(//,4X,'BETA',7X,'R',9X,'Z',9X,'T',8X,'THETA'
         1,5X,'VOL',//)
    900   FORMAT(1H1,43X,'MER.',6X,'CIR.',6X,'MER.',4X,'PRINCIPAL STRESSES'
         1,4X,'MAX',/,29X,'K',5X,'F',7X,'STRESS',4X,'STRESS',4X,'SHEAR'
         2,7X,'S1',8X,'S2',6X,'SHEAR',5X,'ANGLE',//)
    901   FORMAT(26X,I4,8F10.4)
          RETURN
          END
```

APPENDIX D

Balloon Analysis Program - Input

The input for the analysis program consists of three identifying data cards; a deck of manufactured radius, film weight and tape weight; and a deck containing the design shape in nondimensional units.

Card 1. Format (5E15.7)

Columns	Variable	Description
1-15	PD	Design Payload in pounds
16-30	PTOP	Nondimensional top load (P_{TOP}/PD)
31-45	WBAL	Nondimensional balloon weight (W/PD)
46-60	SMAX	Underformed gore length in feet
61-75	LAMBDA	Design $\lambda=(PD/b_d)$**1./3.

Card 2. Format (3E15.7, I15)

1-15	NU	Poissons ratio
16-30	KBAR	Total tape stiffness, NK_T/PD
31-45	EBAR	Film stiffness, $Et\lambda$/PD
46-60	N	Number of gores

Card 3. Format (I15, 2E15.7)

1-15	KMAX	Number of points to be considered
16-30	ABAR	Nondimensional superpressure, a/S_{max}
31-45	BBAR	Ratio of specific lift at altitude to design altitude

Card 4. Format (4E15.7) KMAX Required

Columns	Variable	Description
1-15	GP(K)	Gore position of point K in feet
16-30	RO(K)	Half gore width at point K in inches
31-45	WT(K)	Weight increment between K and K+1 including tape
46-60	WTAPE(K)	Weight increment of a single tape between points K and K+1

Card 5. Format (5E15.7) KMAX Required

1-15	X(L+2)	Normalized design radius at point K, r/λ
16-30	X(L+3)	Normalized design height at point K, z/λ
31-45	X(L+4)	Normalized design load in meridional direction at point K, $T/2\pi PD$
46-60	X(L+5)	Design angle at point K, θ
61-75	X(L+6)	Normalized design volume at point K, $V/\pi\lambda^3$

Output consists of computed payload ratio, normalized deformed shape and stress distributions for any altitude designated as BBAR on card 3. Principal stresses and tension field patterns are computed when necessary. A sample of this output is as follows:

DESIGN VALUES

PAYLOAD= 4000.43 LBS		LAMBDA=165.	EBAR= 82.52
VOLUME= 0.5844D 07 FT(3)		NU= 0.68	ABAR= 0.00
CORE LENGTH= 350.85 FT		KBAR= 25.88	BBAR= 1.00
NUMBER OF TAPES= 91			
NUMBER OF POINTS= 87			

BETA	R	Z	T	THETA	VOL
1.5217	0.0000	0.0000	0.2705	0.9465	0.1353
0.0209	0.0095	0.0069	0.2707	0.9457	0.1353
0.0193	0.0191	0.0137	0.2708	0.9447	0.1353
0.0188	0.0286	0.0206	0.2709	0.9434	0.1353
0.0186	0.0381	0.0275	0.2710	0.9418	0.1353
0.0185	0.0476	0.0344	0.2712	0.9399	0.1352
0.0185	0.0571	0.0413	0.2715	0.9377	0.1352
0.0186	0.0666	0.0482	0.2717	0.9349	0.1351
0.0186	0.0760	0.0551	0.2721	0.9317	0.1350
0.0187	0.0855	0.0621	0.2724	0.9279	0.1348
0.0188	0.0949	0.0691	0.2728	0.9235	0.1347
0.0188	0.1043	0.0761	0.2733	0.9184	0.1344
0.0189	0.1137	0.0831	0.2738	0.9127	0.1342
0.0190	0.1230	0.0902	0.2743	0.9062	0.1339
0.0190	0.1323	0.0974	0.2748	0.8988	0.1335
0.0191	0.1416	0.1046	0.2754	0.8906	0.1331
0.0191	0.1508	0.1119	0.2760	0.8814	0.1326
0.0192	0.1599	0.1192	0.2765	0.8712	0.1320
0.0192	0.1690	0.1266	0.2772	0.8600	0.1314
0.0193	0.1780	0.1341	0.2778	0.8476	0.1307
0.0193	0.1870	0.1418	0.2784	0.8341	0.1299
0.0193	0.1958	0.1495	0.2791	0.8193	0.1290
0.0193	0.2045	0.1573	0.2797	0.8033	0.1280
0.0193	0.2132	0.1653	0.2803	0.7858	0.1270
0.0193	0.2217	0.1733	0.2810	0.7670	0.1258
0.0192	0.2300	0.1816	0.2816	0.7467	0.1244
0.0192	0.2382	0.1900	0.2822	0.7249	0.1230
0.0192	0.2462	0.1985	0.2828	0.7016	0.1214
0.0191	0.2541	0.2072	0.2834	0.6766	0.1197
0.0190	0.2617	0.2161	0.2839	0.6499	0.1179
0.0190	0.2692	0.2251	0.2844	0.6216	0.1159
0.0189	0.2764	0.2344	0.2849	0.5915	0.1137
0.0188	0.2833	0.2438	0.2854	0.5596	0.1113
0.0187	0.2899	0.2535	0.2856	0.5259	0.1088
0.0185	0.2963	0.2633	0.2861	0.4905	0.1062
0.0184	0.3023	0.2733	0.2864	0.4532	0.1033
0.0182	0.3079	0.2836	0.2867	0.4141	0.1003
0.0180	0.3132	0.2940	0.2869	0.3731	0.0971
0.0178	0.3181	0.3046	0.2870	0.3304	0.0937
0.0176	0.3225	0.3154	0.2870	0.2860	0.0901
0.0174	0.3265	0.3264	0.2869	0.2398	0.0864
0.0171	0.3300	0.3375	0.2868	0.1920	0.0825
0.0168	0.3330	0.3488	0.2865	0.1431	0.0785
0.0161	0.3354	0.3602	0.2862	0.0923	0.0744
0.0158	0.3373	0.3717	0.2858	0.0436	0.0702
0.0130	0.3386	0.3832	0.2852	-0.0097	0.0659
0.0129	0.3393	0.3948	0.2845	-0.0488	0.0616
0.0081	0.3396	0.4065	0.2837	-0.1046	0.0573

0.0081	0.3390	0.4181	0.2825	-0.1288	0.0530
0.0183	0.3384	0.4297	0.2817	-0.1868	0.0487
0.0184	0.3365	0.4412	0.2818	-0.2448	0.0445
0.0182	0.3342	0.4526	0.2830	-0.3030	0.0404
0.0177	0.3310	0.4638	0.2830	-0.3613	0.0365
0.0176	0.3273	0.4748	0.2841	-0.4195	0.0327
0.0174	0.3228	0.4856	0.2841	-0.4775	0.0291
0.0174	0.3176	0.4961	0.2848	-0.5350	0.0257
0.0174	0.3119	0.5062	0.2849	-0.5921	0.0225
0.0175	0.3056	0.5160	0.2852	-0.6486	0.0195
0.0177	0.2987	0.5255	0.2853	-0.7043	0.0168
0.0180	0.2913	0.5345	0.2853	-0.7591	0.0144
0.0184	0.2833	0.5431	0.2853	-0.8129	0.0122
0.0189	0.2750	0.5512	0.2854	-0.8657	0.0102
0.0195	0.2661	0.5589	0.2854	-0.9173	0.0084
0.0201	0.2569	0.5661	0.2854	-0.9676	0.0069
0.0208	0.2473	0.5728	0.2854	-1.0165	0.0056
0.0216	0.2374	0.5790	0.2854	-1.0639	0.0045
0.0223	0.2271	0.5847	0.2855	-1.1097	0.0035
0.0232	0.2167	0.5899	0.2855	-1.1539	0.0027
0.0240	0.2059	0.5946	0.2856	-1.1962	0.0021
0.0248	0.1950	0.5989	0.2856	-1.2366	0.0016
0.0256	0.1839	0.6027	0.2857	-1.2750	0.0012
0.0264	0.1727	0.6061	0.2856	-1.3113	0.0008
0.0272	0.1613	0.6090	0.2855	-1.3455	0.0006
0.0279	0.1499	0.6116	0.2853	-1.3773	0.0004
0.0286	0.1384	0.6138	0.2850	-1.4069	0.0003
0.0292	0.1268	0.6157	0.2845	-1.4340	0.0002
0.0298	0.1151	0.6172	0.2837	-1.4586	0.0001
0.0305	0.1034	0.6184	0.2825	-1.4807	0.0001
0.0314	0.0917	0.6194	0.2808	-1.5004	0.0000
0.0331	0.0800	0.6202	0.2786	-1.5174	0.0000
0.0360	0.0683	0.6207	0.2755	-1.5327	0.0000
0.0409	0.0565	0.6211	0.2716	-1.5449	0.0000
0.0468	0.0448	0.6214	0.2673	-1.5564	0.0000
0.0529	0.0330	0.6215	0.2630	-1.5636	-0.0000
0.0564	0.0212	0.6215	0.2605	-1.5711	-0.0000
0.0546	0.0094	0.6215	0.2605	-1.5729	-0.0000
0.0555	0.0000	0.6215	0.2737	-1.5749	0.0000

K	F	MER. STRESS	CIR. STRESS	MER. SHEAR	PRINCIPAL STRESSES S1	S2	MAX SHEAR	ANGLE
1	0.0187	0.8628	0.0008	-0.0241	0.8634	0.0000	0.4314	-0.0279
2	0.0181	0.8380	0.0007	-0.0231	0.8387	0.0001	0.4193	-0.0276
3	0.0176	0.8153	0.0026	-0.0219	0.8159	0.0020	0.4069	-0.0269
4	0.0171	0.7947	0.0055	-0.0207	0.7952	0.0050	0.3951	-0.0262
5	0.0167	0.7758	0.0094	-0.0197	0.7763	0.0089	0.3837	-0.0257
6	0.0162	0.7586	0.0141	-0.0188	0.7591	0.0137	0.3727	-0.0252
7	0.0158	0.7429	0.0198	-0.0180	0.7434	0.0193	0.3620	-0.0249
8	0.0154	0.7286	0.0262	-0.0173	0.7290	0.0258	0.3516	-0.0246
9	0.0150	0.7155	0.0334	-0.0166	0.7159	0.0330	0.3415	-0.0244
10	0.0146	0.7035	0.0412	-0.0161	0.7039	0.0408	0.3315	-0.0243
11	0.0143	0.6925	0.0497	-0.0156	0.6928	0.0493	0.3218	-0.0242
12	0.0139	0.6824	0.0588	-0.0151	0.6827	0.0584	0.3122	-0.0243
13	0.0136	0.6731	0.0683	-0.0147	0.6734	0.0680	0.3027	-0.0243
14	0.0132	0.6645	0.0784	-0.0144	0.6648	0.0780	0.2934	-0.0245
15	0.0129	0.6566	0.0889	-0.0140	0.6569	0.0885	0.2842	-0.0247
16	0.0126	0.6493	0.0997	-0.0137	0.6496	0.0994	0.2751	-0.0249
17	0.0123	0.6426	0.1109	-0.0134	0.6429	0.1105	0.2662	-0.0252
18	0.0120	0.6363	0.1223	-0.0131	0.6366	0.1220	0.2573	-0.0255
19	0.0117	0.6305	0.1340	-0.0129	0.6308	0.1336	0.2486	-0.0259
20	0.0114	0.6251	0.1458	-0.0126	0.6254	0.1455	0.2400	-0.0263
21	0.0111	0.6201	0.1578	-0.0124	0.6204	0.1575	0.2314	-0.0268
22	0.0108	0.6154	0.1700	-0.0121	0.6157	0.1697	0.2230	-0.0272
23	0.0106	0.6110	0.1822	-0.0119	0.6114	0.1819	0.2147	-0.0277
24	0.0103	0.6070	0.1945	-0.0117	0.6073	0.1941	0.2066	-0.0283
25	0.0100	0.6032	0.2068	-0.0114	0.6035	0.2065	0.1985	-0.0288
26	0.0098	0.5997	0.2191	-0.0112	0.6000	0.2188	0.1906	-0.0294
27	0.0095	0.5964	0.2314	-0.0110	0.5967	0.2311	0.1828	-0.0300
28	0.0093	0.5934	0.2437	-0.0107	0.5937	0.2434	0.1751	-0.0307
29	0.0090	0.5906	0.2560	-0.0105	0.5909	0.2557	0.1676	-0.0314
30	0.0088	0.5880	0.2683	-0.0103	0.5884	0.2680	0.1602	-0.0321
31	0.0086	0.5857	0.2805	-0.0101	0.5861	0.2802	0.1529	-0.0329
32	0.0083	0.5837	0.2928	-0.0098	0.5840	0.2925	0.1458	-0.0338
33	0.0081	0.5819	0.3051	-0.0096	0.5822	0.3048	0.1387	-0.0347
34	0.0079	0.5804	0.3174	-0.0094	0.5807	0.3171	0.1318	-0.0357
35	0.0077	0.5791	0.3296	-0.0092	0.5795	0.3295	0.1250	-0.0368
36	0.0075	0.5782	0.3423	-0.0090	0.5785	0.3419	0.1183	-0.0380
37	0.0073	0.5775	0.3549	-0.0088	0.5779	0.3545	0.1117	-0.0393
38	0.0071	0.5772	0.3676	-0.0086	0.5776	0.3673	0.1051	-0.0408
39	0.0069	0.5773	0.3806	-0.0084	0.5776	0.3802	0.0987	-0.0425
40	0.0067	0.5777	0.3938	-0.0082	0.5780	0.3934	0.0923	-0.0444
41	0.0065	0.5785	0.4072	-0.0081	0.5789	0.4069	0.0860	-0.0471
42	0.0064	0.5799	0.4216	-0.0079	0.5803	0.4212	0.0795	-0.0497
43	0.0062	0.5815	0.4356	-0.0093	0.5821	0.4350	0.0735	-0.0634
44	0.0059	0.5870	0.4604	-0.0091	0.5877	0.4598	0.0639	-0.0711
45	0.0056	0.5895	0.4750	-0.0165	0.5918	0.4726	0.0596	-0.1404
46	0.0052	0.6111	0.5470	-0.0161	0.6149	0.5432	0.0358	-0.2328
47	0.0050	0.6142	0.5612	-0.0237	0.6233	0.5521	0.0356	-0.3651

48	0.0041	0.6524	0.6820	-0.0232	0.6947	0.6397	0.0275	2.0727
49	0.0040	0.6558	0.6947	-0.0347	0.7150	0.6554	0.0398	2.1009
50	0.0025	0.7193	0.2769	-0.0321	0.7216	0.2746	0.2235	-0.0720
51	0.0025	0.7238	0.2808	0.0001	0.7238	0.2808	0.2215	0.0002
52	0.0025	0.7331	0.2908	-0.0002	0.7331	0.2908	0.2212	-0.0004
53	0.0025	0.7413	0.3041	0.0001	0.7413	0.3041	0.2186	0.0002
54	0.0025	0.7527	0.3130	0.0006	0.7527	0.3130	0.2198	0.0013
55	0.0026	0.7637	0.3241	0.0008	0.7637	0.3241	0.2198	0.0019
56	0.0026	0.7769	0.3320	0.0013	0.7769	0.3320	0.2224	0.0030
57	0.0026	0.7907	0.3401	0.0017	0.7907	0.3401	0.2253	0.0037
58	0.0027	0.8061	0.3462	0.0022	0.8062	0.3462	0.2300	0.0046
59	0.0028	0.8229	0.3510	0.0027	0.8229	0.3510	0.2360	0.0058
60	0.0028	0.8412	0.3542	0.0033	0.8412	0.3541	0.2436	0.0068
61	0.0029	0.8613	0.3557	0.0039	0.8613	0.3557	0.2528	0.0077
62	0.0030	0.8832	0.3556	0.0045	0.8832	0.3558	0.2637	0.0085
63	0.0031	0.9071	0.3544	0.0051	0.9072	0.3544	0.2764	0.0093
64	0.0033	0.9333	0.3518	0.0055	0.9334	0.3518	0.2908	0.0099
65	0.0034	0.9619	0.3481	0.0064	0.9620	0.3480	0.3070	0.0105
66	0.0036	0.9932	0.3435	0.0071	0.9933	0.3434	0.3250	0.0110
67	0.0038	1.0275	0.3381	0.0079	1.0276	0.3380	0.3448	0.0114
68	0.0040	1.0650	0.3321	0.0087	1.0651	0.3320	0.3665	0.0118
69	0.0042	1.1061	0.3257	0.0095	1.1062	0.3256	0.3903	0.0122
70	0.0044	1.1512	0.3190	0.0104	1.1514	0.3188	0.4163	0.0125
71	0.0047	1.2008	0.3120	0.0114	1.2009	0.3119	0.4445	0.0128
72	0.0049	1.2553	0.3050	0.0125	1.2554	0.3048	0.4753	0.0131
73	0.0052	1.3153	0.2980	0.0136	1.3155	0.2979	0.5088	0.0134
74	0.0056	1.3817	0.2914	0.0149	1.3819	0.2912	0.5453	0.0137
75	0.0059	1.4552	0.2856	0.0164	1.4554	0.2854	0.5850	0.0140
76	0.0063	1.5371	0.2814	0.0179	1.5373	0.2811	0.6281	0.0142
77	0.0068	1.6287	0.2802	0.0195	1.6290	0.2799	0.6746	0.0145
78	0.0072	1.7321	0.2837	0.0213	1.7324	0.2834	0.7245	0.0147
79	0.0078	1.8497	0.2953	0.0234	1.8501	0.2949	0.7776	0.0150
80	0.0083	1.9840	0.3142	0.0259	1.9844	0.3138	0.8353	0.0155
81	0.0090	2.1399	0.3450	0.0299	2.1404	0.3446	0.8979	0.0166
82	0.0097	2.3167	0.3694	0.0362	2.3174	0.3687	0.9743	0.0186
83	0.0107	2.5256	0.3857	0.0466	2.5267	0.3846	1.0710	0.0218
84	0.0119	2.7675	0.3551	0.0635	2.7691	0.3534	1.2079	0.0263
85	0.0136	3.0802	0.2780	0.0885	3.0830	0.2752	1.4039	0.0315
86	0.0160	3.4906	0.1324	0.1279	3.4954	0.1275	1.6840	0.0380
87	0.0196	4.1551	0.0000	0.1775	4.1626	-0.0076	2.0851	0.0426